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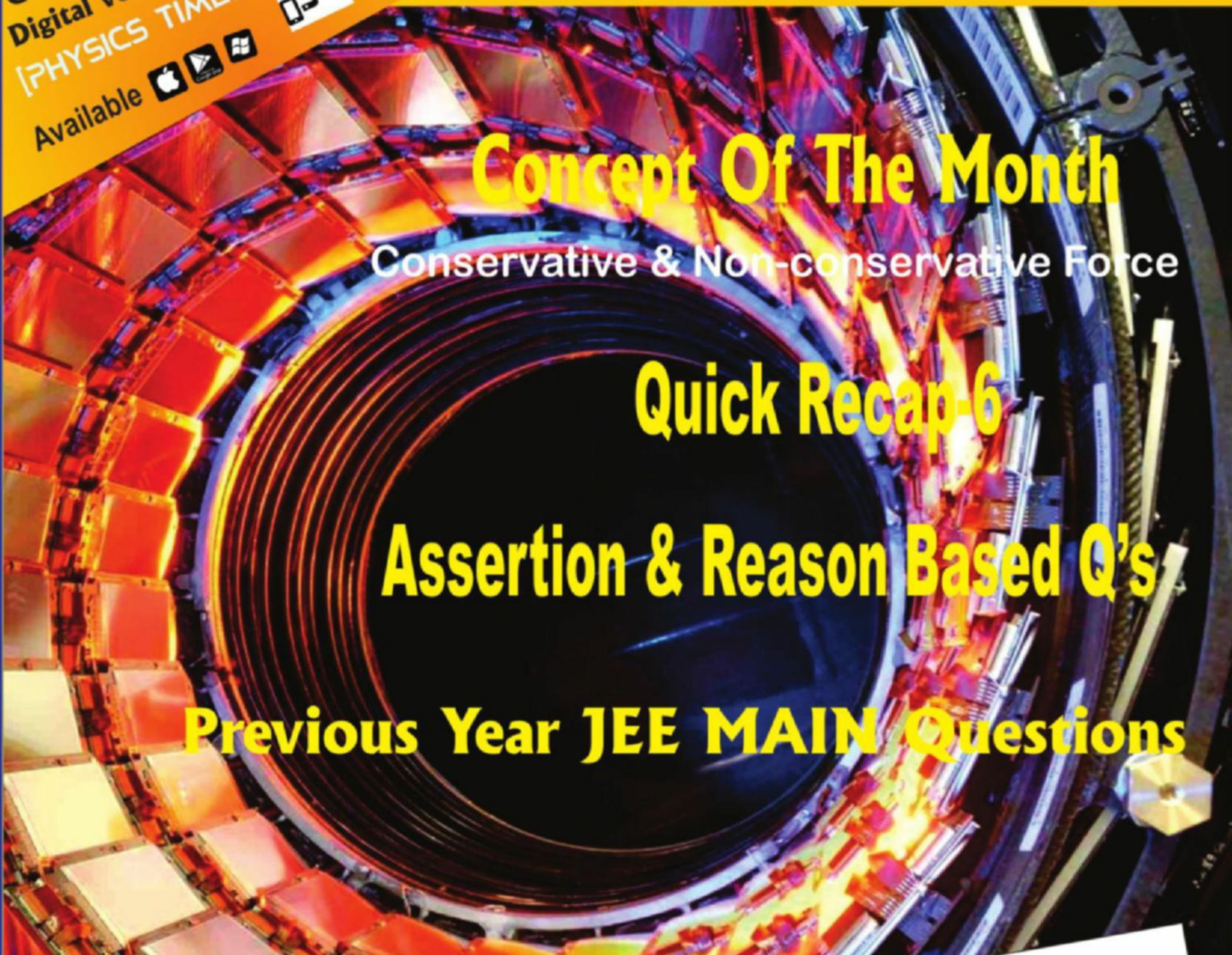

Concept Of The Month

Conservative & Non-conservative Force

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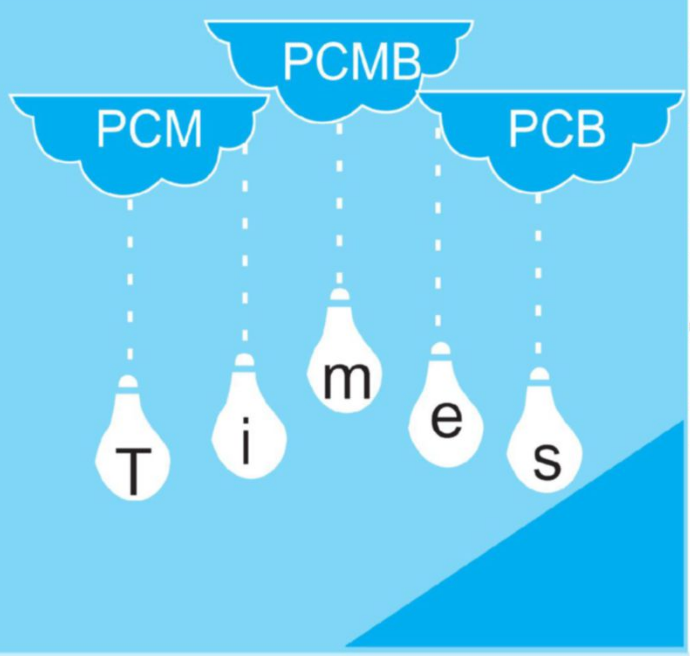

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CONTENTS

Conservative and Non-Conservative Forces

Concept of the month

This column is aimed at preparing students for all competitive exams like JEE ADVANCED, BITSAT etc. Every concept has been designed by highly qualified faculty to cater to the needs of the students by discussing the most complicated and confusing concepts in Physics.

By: ESWAR REDDY ALLA (Bangalore)
B. MADHU (Bangalore)

Introduction

Forces in nature can be divided into two types:

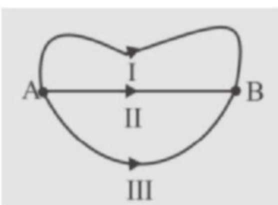
(i) Conservative force

(ii) Nonconservative force

In this article we have discussed two different methods to verify a given force as conservative or non-conservative. In one method we assume a closed path of our choice and find the work done in that path. The second method is a short-cut method.

(i) Conservative force

- If work done by the force around a closed path is zero and it is independent of path then it is a conservative force.
- Consider two points A and B. A body can be displaced by any of the three paths shown in the figure.



If $\underset{\text{Path I}}{W_{A\to B}} = \underset{\text{Path II}}{W_{A\to B}} = \underset{\text{Path III}}{W_{A\to B}}$

$$\underset{\text{Path I}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path II}}{\int \vec{F}.d\vec{l}} = \underset{\text{Path III}}{\int \vec{F}.d\vec{l}}$$

- As work is independent of the path then the force is conservative.

Eg : Gravitational force, Elastic force, Electrostatic force etc.

- All central forces are conservative forces.
- A conservative force is only a function of position not of velocity or time.
- Under conservative force, $F = -\frac{dU}{dx}$ where U is Potential Energy.
- Change in potential energy, $dU = -\int \vec{F}\cdot d\vec{x}$
- A conservative force is always related to potential energy as

$$\vec{F} = -\vec{\nabla}U, \quad \vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Where ∇ is called as dell

The property of a conservative force is

$$\vec{\nabla}\times\vec{F} = \vec{\nabla}\times -\vec{\nabla}U = -\left(\vec{\nabla}\times\vec{\nabla}\right)U = -(0)U = 0$$

Where $\vec{\nabla}\times\vec{F}$ is also called as Curl F

$$\vec{\nabla}\times\vec{F} = 0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = 0$$

$$\hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = 0$$

$$\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} = 0 \Rightarrow \frac{\partial F_z}{\partial y} = \frac{\partial F_y}{\partial z} \to (\text{i})$$

Similarly we get

$$\frac{\partial F_z}{\partial x}=\frac{\partial F_x}{\partial z}\rightarrow(\text{ii}) \quad \& \quad \frac{\partial F_y}{\partial x}=\frac{\partial F_x}{\partial y}\rightarrow(\text{iii})$$

For a given force vector $\overline{F}=F_x\hat{i}+F_y\hat{j}+F_z\hat{k}$ the force is said to be conservative if it satisfies all the three equations. For a two dimensional force $\overline{F}=F_x\hat{i}+F_y\hat{j}$ the force is said to be conservative if it satisfies the 3rd equation

$$\frac{\partial F_y}{\partial x}=\frac{\partial F_x}{\partial y} \quad (\because F_z=0).$$

(ii) Non-conservative force:

The work done by a non conservative force depends on the path followed by a body.

e.g: Friction, air resistance etc..

1. Verify whether the given force is conservative or not

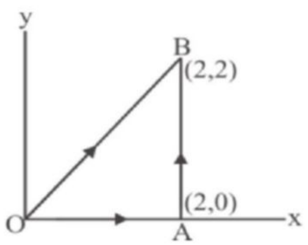
$$\overline{F}=xy^2\hat{i}+yx^2\hat{j}$$

1.Sol: Method-I

If the work done by a force is independent of the path followed by the body then it is a conservative force.

Consider any closed path. For simplicity consider a triangle OAB as shown in the figure.

If the work done along $O\rightarrow A\rightarrow B$ and $O\rightarrow B$ are equal then the given force is conservative.



Here the coordinates of A and B can be taken anywhere.

Path (OA)

Along OA x - changes from 0 to 2

$\Rightarrow dx \neq 0$

y - remains constant

$y=0\Rightarrow dy=0$

$$W_{OA}=\int F_x dx+\int F_y dy$$

$$=\int_{x=0}^{x=2} xy^2 dx+\int_{y=0}^{y=0} yx^2 dy$$

$$=\int x(0)^2 dx+\int 0(x^2)0$$

$$=0+0=0$$

Path (AB)

Along AB x = constant = 2 $\Rightarrow dx=0$

$y-$ changes from $0\rightarrow 2$

$dy\neq 0$

$$W_{AB}=\int F_x dx+\int F_y dy$$

$$=\int_{x=2}^{x=2} xy^2 dx+\int_{y=0}^{y=2} yx^2 dy$$

$$=\int 2(y)^2(0)+\int_0^2 y(2^2)dy$$

$$=4\int_0^2 ydy=4\left[\frac{y^2}{2}\right]_0^2=4\times\frac{4}{2}=8$$

$$W_{O\rightarrow A\rightarrow B}=8.$$

Path (OB)

OB is a straight line

$x-$ changes from $0\rightarrow 2 \quad dx\neq 0$

$y-$ changes from $0\rightarrow 2 \quad dy\neq 0$

The relation between x and y is

$$y=mx \;\Rightarrow y=x \quad \left(m=\frac{2-0}{2-0}=1\right)$$

$$W_{OB}=\int F_x dx+\int F_y dy$$

$$=\int_{x=0}^{x=2} xy^2 dx+\int_{y=0}^{y=2} yx^2 dy$$

$$=\int_0^2 x^3 dx+\int_0^2 y^3 dy$$

$$=\left[\frac{x^4}{4}\right]_0^2+\left[\frac{y^4}{4}\right]_0^2$$

$$\left[\frac{2^4}{4}-0\right]+\left[\frac{2^4}{4}-0\right]=8$$

$W_{OAB}=W_{OB}$

The given force is conservative.

(Method-II)

The given force $\overline{F}=F_x\hat{i}+F_y\hat{j}+F_z\hat{k}$ is said to be conservative if it satisfies the following three equations,

$$\frac{\partial F_x}{\partial y}=\frac{\partial F_y}{\partial x} \quad (1)$$

$$\frac{\partial F_y}{\partial z}=\frac{\partial F_Z}{\partial y} \quad (2)$$

$$\frac{\partial F_z}{\partial x}=\frac{\partial F_x}{\partial z} \quad (3)$$

For a two dimensional force

$\overline{F}=F_x\hat{i}+F_y\hat{j}$ then the force is conservative if it satisfies eq (1)

Given $\overline{F}=xy^2\hat{i}+yx^2\hat{j}$

$F_x=xy^2 \qquad F_y=x^2y$

$$\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial y}(xy^2)$$

$$=x\frac{\partial}{\partial y}(y^2)$$

$$=x(2y)=2xy$$

$$\frac{\partial F_y}{\partial x}=\frac{\partial}{\partial x}(yx^2)$$

$$=y\frac{\partial}{\partial y}(x^2)=y(2x)=2xy$$

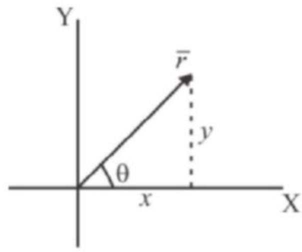
The given force is conservative

2. Consider the gravitation force between two masses m and M

$$\overline{F}=\frac{GmM}{r^2}\hat{r}=\frac{GmM}{r^3}\overline{r}$$

Where $\overline{r}$ is the radial vector drawn from the centre of M to the centre of m. Verify whether it is a conservative or not.

2.Sol : $\overline{F}=\frac{GmM}{r^2}\hat{r}$



$\overline{r}=x\hat{i}+y\hat{j}$

$r=\sqrt{x^2+y^2}$

$\hat{r}=\cos\theta\hat{i}+\sin\theta\hat{j}$

$\hat{r}=\frac{x}{r}\hat{i}+\frac{y}{r}\hat{j}$

$$\overline{F}=\frac{C}{\left(\sqrt{x^2+y^2}\right)^2}\left(\frac{x}{r}\hat{i}+\frac{y}{r}\hat{j}\right)$$

(where C = GmM)

$$\overline{F}=\frac{Cx}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{Cy}{\left(x^2+y^2\right)^{3/2}}\hat{j}$$

$$F_x=\frac{Cx}{\left(x^2+y^2\right)^{3/2}} \qquad F_y=\frac{Cy}{\left(x^2+y^2\right)^{3/2}}$$

$$\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial y}\left(\frac{Cx}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$=Cx\frac{\partial}{\partial y}\left(x^2+y^2\right)^{-3/2}$$

$$=-\frac{3}{2}Cx\left(x^2+y^2\right)^{-5/2}(2y)$$

$$\frac{\partial F_x}{\partial y}=-3Cxy\left(x^2+y^2\right)^{-5/2}$$

$$\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial x}\left(\frac{Cy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$=Cy\frac{\partial}{\partial x}\left(x^2+y^2\right)^{-3/2}$$

$$= -\frac{3}{2}Cy\left(x^2+y^2\right)^{-5/2}\frac{\partial}{\partial x}\left(x^2+y^2\right)$$

$$= -\frac{3}{2}Cy\left(x^2+y^2\right)^{-5/2}(2x)$$

$$\frac{\partial F_y}{\partial x} = -3Cxy\left(x^2+y^2\right)^{-5/2}$$

As $\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$ the gravitational force is conservative.

Exercise

1. When a conservative force does positive work on a body:
(a) The potential energy increases
(b) The potential energy decreases
(c) Total energy increases
(d) Total energy decreases

2. Which one of the following is not a conservative force?
(a) Electrostatic force (b) Magnetic force
(c) Friction (d) Gravitational force

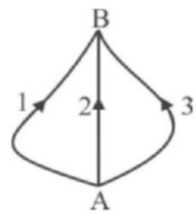
3. In which of the following cases can the work done increase the potential energy?
(a) Conservative force only
(b) Non-conservative force only
(c) Neither conservative nor non-conservative forces
(d) Both conservative and non-conservative forces

4. Work done by a conservative force
(a) Never depends on path
(b) May depend on path
(c) Is not related to potential energy
(d) None of these

5. Work done by the conservative forces on a system is equal to
(a) The change in kinetic energy of the system
(b) The (-ve) of change in potential energy of the system
(c) The (-ve) change in total mechanical energy of the system
(d) None of the above

6. Which of the following statements are correct?
A. If a particle moves opposite to the conservative field, potential energy will increases
B. If a particle moves in the direction of conservative field, the potential energy will increase
C. If a particle moves opposite to the conservative field, work done by the field will be positive
D. If a particle moves opposite to the conserv ative field, work done by the field will be negative
(a) A, C (b) A, D
(c) B, C (d) B, D

7. If W_1, W_2 *and* W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 respectively in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 *and* W_3.



(a) $W_1 = W_2 = W_3$ (b) $W_1 < W_2 < W_3$
(c) $W_1 > W_2 > W_3$ (d) $W_1 < W_2 > W_3$

8. Negative of work done by the conservative forces on a system is equal to :
(a) The change in kinetic energy of the system
(b) The change in potential energy of the system
(c) The change in total mechanical energy of the system
(d) None of these

9. Which of the following force is conservative
(a) $5\hat{j}$ (b) $-6\hat{k}$
(c) $2\hat{i}$ (d) All of the above

10. Which of the following is a conservative force?
(a) $xy^2\hat{i} + x^2y\hat{j}$ (b) $xy\,\hat{i} + xy\,\hat{j}$

(c) $y^2\hat{i}+x^2\hat{j}$ (d) $y\hat{i}+x^2\hat{j}$

11. Verify whether the following forces are conservative or not.

(1) $\overline{F}=xy\hat{i}+xy\hat{j}$ Ans : Non conservative

(2) $\overline{F}=x\hat{i}+y\hat{j}$ Ans : conservative

(3) $\overline{F}=\frac{\hat{r}}{r^3}$ Ans : conservative

(4) $\overline{F}=r^2\hat{r}$ Ans : conservative

(5) $\overline{F}=C\hat{i}$ (C - is a constant)

Ans : conservative

ANSWER KEY

1. b	2. c	3. a	4. a	5. b
6. b	7. a	8. b	9. d	10. a

HINTS & SOLUTIONS

1.Sol: Positive work done by a conservative force always decreases the potential energy

$W=-\Delta U$

$\Rightarrow \Delta U=-(W)=-(+ve)=-ve$

2.Sol: Among the given forces, force of friction is a non-conservative force whereas all other forces are conservative forces.

3.Sol: In case of non-conservative forces, the work done is dissipated as heat, sound etc, i.e., it does not increase the potential energy.

4.Sol: Conceptual

5.Sol: $W=-\Delta U$

6.Sol: Conceptual

7.Sol: Gravitational force is a conservative force and work done against it does not depend on the path.

8.Sol: Conceptual

9.Sol: Any constant force is a conservative force.

10.Sol: If the force $\vec{F}=F_x\hat{i}+F_y\hat{j}$ is conservative then

$$\frac{\partial F_x}{\partial y}=\frac{\partial F_y}{\partial x}$$

$$\vec{F}=xy^2\hat{i}+x^2y\hat{j}$$

$$F_x=xy^2, F_y=x^2y$$

$$\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial y}(xy^2)=2xy$$

$$\frac{\partial F_y}{\partial x}=\frac{\partial}{\partial x}(x^2y)=2xy$$

$$\Rightarrow \frac{\partial F_x}{\partial y}=\frac{\partial F_y}{\partial x}$$

$$\vec{F}=xy\hat{i}+xy\hat{j}$$

$$F_x=xy, F_y=xy$$

$$\frac{\partial F_x}{\partial y}=x \; \& \; \frac{\partial F_y}{\partial x}=y$$

$$\Rightarrow \frac{\partial F_x}{\partial y}\neq\frac{\partial F_y}{\partial x}$$

$$\vec{F}=y^2\hat{i}+x^2\hat{j}$$

$$F_x=y^2, F_y=x^2$$

$$\frac{\partial F_x}{\partial y}=\frac{\partial}{\partial y}(y^2)=2y$$

$$\frac{\partial F_y}{\partial x}=\frac{\partial}{\partial x}(x^2)=2x$$

$$\frac{\partial F_y}{\partial x}\neq\frac{\partial F_x}{\partial y}$$

$$\vec{F}=y\hat{i}+x^2\hat{j}$$

$$F_x=y, F_y=x^2$$

$$\frac{\partial F_x}{\partial y}=1, \frac{\partial F_y}{\partial x}=2x$$

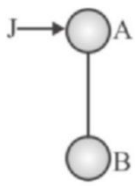
$$\frac{\partial F_x}{\partial y}\neq\frac{\partial F_y}{\partial x}$$

So option (a) is correct.

GLIMPSE OF ELITE SERIES

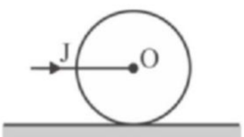
ANGULAR IMPULSE

1. Two particles of equal mass m at A and B are connected by a rigid light rod AB lying on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular to AB. The velocity of particle at A is:



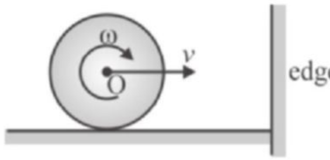
(a) $\frac{2J}{m}$ (b) Zero (c) $\frac{J}{m}$ (d) $\frac{J}{2m}$

2. An impulse J is applied on a ring of mass m along a line passing through its centre O. The ring is placed on a rough horizontal surface. The linear velocity of centre of ring once it starts rolling without slipping is



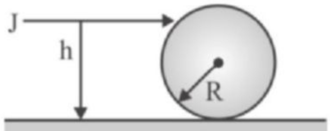
(a) J/3m (b) J/m (c) J/4m (d) J/2m

3. A uniform solid sphere of radius r is rolling on a smooth horizontal surface with velocity v and angular velocity $\omega\left(v=\omega r\right)$. The sphere collides with the wall as shown in the figure. The coefficient of friction between the sphere and the edge $\mu = 1/5$. Just after the collision the angular velocity of the sphere becomes zero. The linear velocity of the sphere just after the collision is equal to



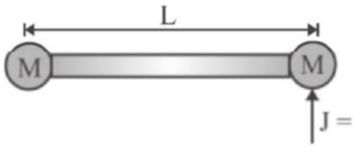
(a) $\frac{v}{6}$ (b) $\frac{v}{5}$ (c) v (d) $\frac{3v}{5}$

4. A solid sphere of mass M and radius R is placed on a rough horizontal surface. It is struck by a horizontal cue stick at a height h above the surface. The value of h so that the sphere performs pure rolling motion immediately after it has been struck is



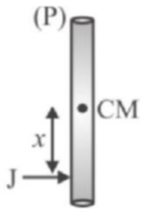
(a) $\frac{7R}{5}$ (b) $\frac{2R}{5}$ (c) $\frac{9R}{5}$ (d) $\frac{5R}{5}$

5. Consider a body, shown in the figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse J = Mv is imparted to the body at one of its ends, what would be its angular velocity?



(a) V/L (b) 2V/L (c) V/3L (d) V/4L

6. An impulse J is exerted on a rod of mass m and length L at a distance x from the C.M. Find the value of x at which the net velocity of point P is zero.



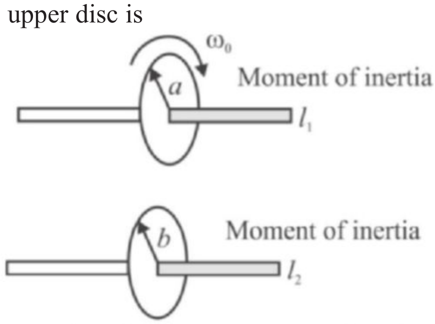
(a) $\frac{L}{12}$ (b) $\frac{L}{8}$ (c) $\frac{L}{3}$ (d) $\frac{L}{6}$

7. A cockroach of mass m is moving on the rim of a disc with velocity v in the anticlockwise direction. The moment of inertia of the disc about its own axis is I and it is rotating in the clockwise direction with angular speed ω. If the cockroach stops moving then the angular speed of the disc will be

(a) $\frac{I\omega - mvR}{I}$ (b) $\frac{I\omega + mvR}{I + mR^2}$

(c) $\frac{I\omega - mvR}{I + mR^2}$ (d) $\frac{I\omega}{I + mR^2}$

8. The two uniform discs rotate separately on parallel axles. The upper disc (radius a and momentum of inertia I_1) is given an angular velocity ω_0 and the lower disc of (radius b and momentum of inertia I_2) is at rest. Now the two discs are moved together so that their rims touch. Final angular velocity of the upper disc is



(a) $\frac{(I_1\omega_0)}{\left[I_1 + (a^2 I_2 / b^2)\right]}$ (b) $\frac{(I_1\omega_0)}{\left[I_1 + (b^2 I_2 / a^2)\right]}$

(c) $\frac{(I_2\omega_0)}{\left[I_2 + (b^2 I_1 / a^2)\right]}$ (d) $\frac{(I_2\omega_0)}{\left[I_2 + (a^2 I_1 / b^2)\right]}$

ANSWER KEY

1. c	2. d	3. c	4. a	5. a
6. d	7. c	8. a		

HINTS & SOLUTIONS

1.Sol: Let l is the length of the rod.

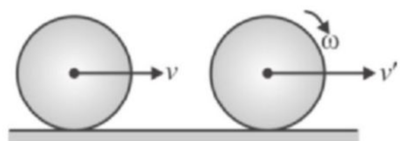
$$v = \frac{J}{2m}$$

$$J\left(\frac{l}{2}\right) = \left[m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2\right]\omega$$

$$\omega = \frac{J}{ml}$$

$$\therefore v_A = v + \frac{l}{2}\omega = \frac{J}{2m} + \frac{J}{2m} = \frac{J}{m}$$

2.Sol: Let v be the velocity of COM of ring just after the impulse is applied and v' its velocity when pure rolling starts. Angular velocity ω of the ring at this instant will be $\omega = \frac{v'}{r}$.



From impulse = change in linear momentum, we have

$$J = mv \Rightarrow v = J/m$$

Between the two positions shown in the figure, force of friction on the ring acts backwards. Angular momentum of the ring about bottom most point will remain conserved

$$\therefore L_i = L_f$$

$$mvr = mv'r + I\omega$$

$$mv'r + (mr^2)(v'/r) = 2mv'r$$

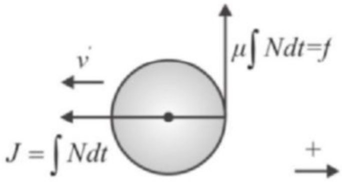
$$v' = \frac{v}{2} = \frac{J}{2m}$$

3.Sol: Impulse provided by the edge in the horizontal direction:

$$\int N dt = mv' - (-mv) \qquad (i)$$

Angular impulse by friction in the vertical direction

$$fR = \mu R \int N dt = \frac{2}{5} mR^2 \left(\frac{v}{R}\right) \qquad (ii)$$



From eqs. (i) and (ii), we get

$$\int N dt = 2mV \text{ and } v' = v$$

4.Sol: Let v be the velocity of the centre of mass of the sphere and ω be the angular velocity of the body about an axis passing through the centre of mass.

The linear impulse is

J = Mv

The angular impulse is

$$J(h-R) = \frac{2}{5} MR^2 \times \omega$$

From the above two equations, $v(h-R) = \frac{2}{5} R^2 \omega$

From the condition of pure rolling, $v = R\omega$

$$h - R = \frac{2R}{5} \Rightarrow h = \frac{7R}{5}$$

5.Sol: Given system of two particles will rotate about its centre of mass.

Initial angular momentum $= Mv\left(\frac{L}{2}\right)$

Final angular momentum $= 2M\left(\frac{L}{2}\right)^2 \omega$

From conservation of angular momentum

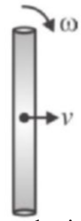
$$Mv\left(\frac{L}{2}\right) = 2M\left(\frac{L}{2}\right)^2 \omega \Rightarrow \omega = \frac{v}{L}$$

6.Sol: From linear impluse

$J = mv$ (v-velocity of CM)

From angular impluse

$$Jx = \frac{mL^2}{12} \omega$$



As the point P velocity is zero.

$$v_P = v - \frac{l}{2}\omega = 0$$

$$v = \frac{l}{2}\omega$$

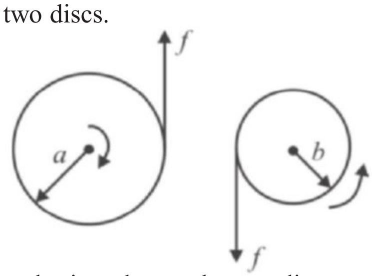
From the above three eq's $x = \frac{L}{6}$

7.Sol: $L_i = I\omega - mvR$

$$L_f = \left(I + mR^2\right)\omega'$$

$$L_f = L_i \Rightarrow \omega' = \frac{I\omega - mvR}{I + mR^2}$$

8.Sol: The two discs exert equal and opposite forces on each other when in contact. The torque due to these forces changes the angular momentum of each disc. Let ω_1 and ω_2 are the angular velocities of the two discs.



The angular impulse on the two discs are

$$fa\Delta t = I_1(\omega_0 - \omega_1) \qquad (i)$$

and $fb\Delta t = I_2\omega_2$ (ii)

From eqns. (i) and (ii), we get

$$\frac{a}{b} = \frac{I_1(\omega_0 - \omega_1)}{I_2\omega_2} \qquad (iii)$$

When slipping ceases between the discs, the contact points of the two discs have the same linear velocity, i.e.,

$$a\omega_1 = b\omega_2 \qquad (iv)$$

On substituting ω_2 in eq's (iii), we get

$$\omega_1 = \frac{(I_1\omega_0)}{\left[I_1 + \left(a^2 I_2 / b^2\right)\right]}$$

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PAPER-1

SECTION-1
Single answer type questions

1. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with small area $\alpha 4\pi R^2 (\alpha << 1)$ is made on the shell without affecting the rest of the shell. Which one of the following statement is correct.
(a) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$
(b) The potential at the centre of shell is reduced by $2\alpha V_0$
(c) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$
(d) The magnitude of electric field at a point, located on a line passing through the hole and shell's center, on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

2. In a radioactive sample ${}^{40}_{19}K$ nuclei either decay into stable ${}^{40}_{20}Ca$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}Ar$ nuclei with decay constant 0.5×10^{-10} per year.
Given that in this sample all the stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei are produced by the ${}^{40}_{19}K$ nuclei only. In time t × 10^9 years, if the ratio of the sum of stable ${}^{40}_{20}Ca$ and ${}^{40}_{18}Ar$ nuclei to the radioactive ${}^{40}_{19}K$ nuclei is 99, the value of t will be. [Given : $\ln 10 = 2.3$]
(a) 9.2 (c) 1.15 (c) 4.6 (d) 2.3

3. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as
$$T(t) = T_0(1+\beta t^{1/4})$$
where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature.
The heat capacity of metal is:
(a) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (b) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$
(c) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (d) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

4. Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \rho(r)/m$ is : (G = universal gravitational constant)

(a) $\dfrac{K}{6\pi r^2 m^2 G}$ (b) $\dfrac{K}{\pi r^2 m^2 G}$

(c) $\dfrac{3K}{\pi r^2 m^2 G}$ (d) $\dfrac{K}{2\pi r^2 m^2 G}$

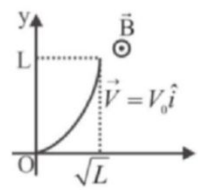
SECTION-2

More than one answer type questions

1. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following in statement(s) is/are correct?
(a) The dimension of force is L^{-3}
(b) The dimension of power is L^{-5}
(c) The dimension of energy is L^{-2}
(d) The dimension of linear momentum is L^{-1}

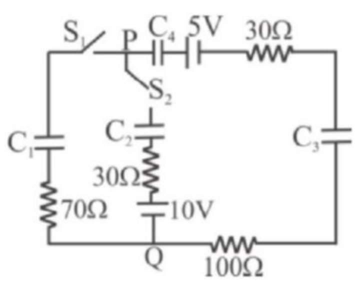
2. Two identical moving coil galvanometers have 10Ω resistance and full scale deflection at $2\mu A$ current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with $R = 1000\Omega$ resistor by using an ideal cell. Which of the following statement(s) is/are correct ?
(a) The resistance of the Voltmeter will be $100\,k\Omega$
(b) The resistance of the Ammeter will be $0.02\ \Omega$ (round off to 2nd decimal place)
(c) If the ideal cell is replaced by a cell having internal resistance of $5\ \Omega$ then the measured value of R will be more than $1000\ \Omega$
(d) The measured value of R will be $978\ \Omega < R < 982\Omega$

3. A conducting wire of parabolic shape, $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1 + \left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$ as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statements(s) is/are :



(a) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$
(b) $|\Delta\phi|$ remains same if the parabolic wire is replaced by a straight wire, y = x, initially, of length $\sqrt{2}l$
(c) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$
(d) $|\Delta\phi|$ is proportional to the length of wire projected on y-axis

4. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F, C_2 = 30\mu F$ and $C_3 = C_4 = 80\mu F$. Which statements is/are correct :



(a) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time the instantaneous current across 30Ω resistor (between points P & Q) will be 0.2A (round off to 1st decimal place).

(b) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across C_1 will be 4V.

(c) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA

(d) if S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between P and Q will be 10V.

5. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces.
Which of the following option(s) is are correct
[ε_0 is the permittivity of free space]

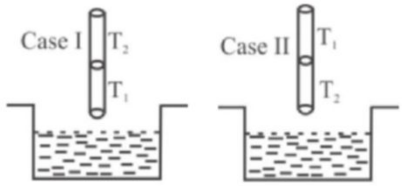
(a) If $h > 2R$ and $r = \frac{4R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

(b) If $h > 2R$ and $r = \frac{3R}{5}$ then $\phi = \frac{Q}{5\varepsilon_0}$

(c) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\phi = 0$

(d) If $h > 2R$ and $r > R$ then $\phi = \frac{Q}{\varepsilon_0}$

6. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60° respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?
[Surface tension of water = 0.075N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



(a) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(b) For case I, capillary joint is 5cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(c) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(d) For case II, the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

7. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal. f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and ($1 < n < 2$), the correct statement(s) is/are

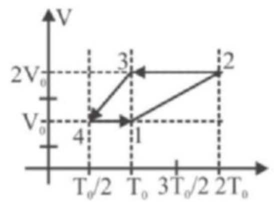
(a) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

(b) For n = 1.5, $\Delta n = 10^{-3}$ and $f = 20\,\text{cm}$, the value of $|\Delta f|$ will be 0.02 cm (round off to 2nd decimal place).

(c) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(d) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

8. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are : [R is the gas constant)



(a) Work done in this thermodynamic cycle $(1 \to 2 \to 3 \to 4 \to 1)$ is $|W| = \frac{1}{2}RT_0$

(b) The ratio of heat transfer during processes $1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(c) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(d) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{1}{2}$

SECTION-3

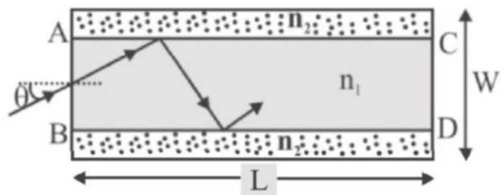
Integer Type Questions

1. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m[th] s layer is $K_m = K\left(1 + \frac{m}{N}\right)$. For a very large N(>$10^3$), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d \ln 2}\right)$. The value of α will be _____ .

[$\in_0$ is the permittivity of free space]

2. A planar structure of length L and width W is made of two different optical media of refractive indices n_1 = 1.5 and n_2 = 1.44 as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is _____ .

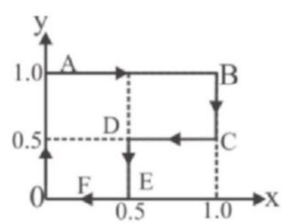
[Speed of light c = 3 × 10^8 m/s]



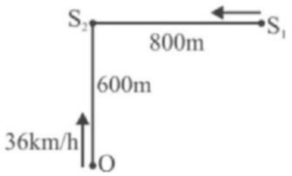
3. A liquid at 30° C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates.

After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _____ °C. (Neglect the heat exchange with surrounding]

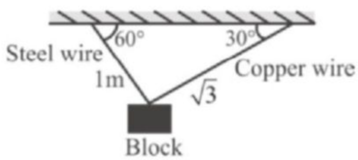
4. A particle is moved along a path AB–BC–CD–DE–EF–FA, as shown in figure in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$, where x and y are in meter and $\alpha w = -1 Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ____joule



5. A train S_1, moving with a uniform velocity of 108 km/h, approaches another train S_2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S_2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S_2 and distance between S_1 and S_2 is 800 m, the number of beats heard by O is _______ .
(Speed of the sound = 330 m/s)



6. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm² and, length $\sqrt{3}m$ and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δlc) and elongation in steel wire is (Δls), then the ratio $\frac{\Delta l_C}{\Delta l_S}$ is ___ .
(Young's modulus for copper and steel are 1 × 10^{11} N/m² and 2 × 10^{11} N/m², respectively)



ANSWER KEY

SECTION-1

1. a 2. a 3. b 4. d

SECTION-2

1. (a,c&d) 2. (b, d) 3. (a,b&d) 4. (b, c) 5. (b,c,d) 6. (a,c&d) 7. (a,b&d) 8. (a,b)

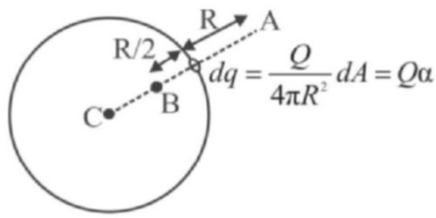
SECTION-3

1. 1.00 2. 50.00 3. 270.00, 120.00 4. 0.75 5. 8.12 or 8.13 6. 2.00

HINTS & SOLUTIONS

SECTION-1

1.Sol:



Let Q is the charge on the shell.
Given that Potential at the surface is

$$V_0 = \frac{KQ}{R}$$

Potential at C

$$V_C = \frac{KQ}{R} - \frac{K(\alpha Q)}{R} = V_0(1-\alpha)$$

Potential at B

$$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1-2\alpha)$$

$$\therefore \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$$

Field at A

$$E_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$

So reduced by $\frac{\alpha V_0}{R}$

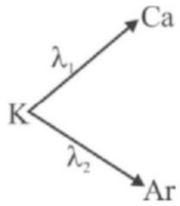
Field at C

As Field by the total shell at C is zero.
So we can assume the hole as a combination of dq and -dq. So the net Filed is only due to -dq.

$$E_C = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$$

So increased by $\frac{\alpha V_0}{R}$

2.Sol:



At $t=0$

$$\frac{dN}{dt}=-(\lambda_1+\lambda_2)t$$

on integrating

$$\log_e\left(\frac{N}{N_0}\right)=-(\lambda_1+\lambda_2)t$$

Given that

$$\frac{N_0-N}{N}=99 \Rightarrow N_0=100N$$

$$2.3\times\log_{10}\left(\frac{N}{N\times100}\right)=-5\times10^{-10}t$$

$$t=9.2\times10^{9}\text{ Year}$$

3.Sol: $dQ=Cdt$

where C is the heat capacity per unit mass

$$\frac{dQ}{dt}=C.\frac{dT}{dt}$$

$$P=C\cdot T_0\cdot\beta\cdot\frac{1}{4}\cdot t^{-3/4}$$

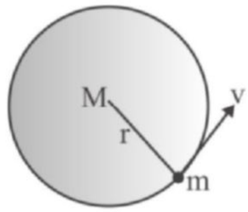
$$\frac{4P}{T_0.\beta}=t^{-3/4}\cdot C$$

Now $\quad T-T_0=T_0\beta t^{1/4}$

$$\Rightarrow \quad t^{3/4}=\left(\frac{T-T_0}{T_0\beta}\right)^3$$

$$\Rightarrow \quad C=\frac{4P(T-T_0)^3}{T_0^4\beta^4}$$

4.Sol: Let M is the mass of the sphere having radius r



$$\frac{GMm}{r^2}=\frac{mv^2}{r}=\frac{2}{r}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad \frac{GMm}{r^2}=\frac{2K}{r} \Rightarrow M=\frac{2Kr}{Gm}$$

$$\Rightarrow \quad dM=\frac{2K}{Gm}dr \Rightarrow 4\pi r^2 dr\rho=\frac{2K}{Gm}dr$$

$$\therefore \quad \rho=\frac{K}{2\pi Gmr^2}$$

$$\Rightarrow \quad \frac{\rho}{m}=\frac{K}{2\pi Gm^2r^2}$$

SECTION-2

1.Sol: Given that $[M]=[Mass]=[M^0L^0T^0]$

[J] = [Angular momentum] $=[ML^2T^{-1}]$

$[L]=[\text{Length}]$

Now; $[M^0L^2T^{-1}]=[\text{Dimensionless quantity}]$

$\therefore \quad [L^2]=[T]$

Power [P] = $[M^0LT^{-2}.LT^{-1}]$

$=[M^0L^2T^{-3}]$

$[P]=[L^{-4}]$

Energy = $[M^0LT^{-2}\cdot L]$

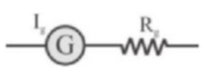
$=[L^2L^{-4}]=[L^{-2}]$

Force [F] $=[F]=[M^0LT^{-2}]=[L.L^{-4}]=[L^{-3}]$

Linear momentum $[p]=[M^0LT^{-1}]=[L.L^{-2}]$

$[p]=[L^{-1}]$

2.Sol:

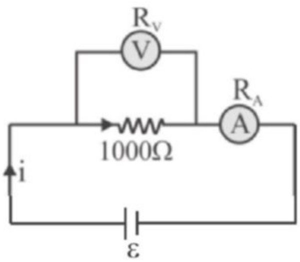


$$V=100\times10^{-3}V=10^{-1}V$$

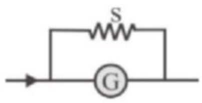
$$V=I_g(R_g+R_V)$$

$$\frac{10^{-1}}{2\times10^{-6}} = R_g + R_V$$

$$5\times10^{4}\,\Omega \approx R_V \qquad (R_g < R_V)$$



$$I_g R_g = (I - I_g)S$$



$$S = \frac{2\times10^{-6}\times10}{10^{-3} - 2\times10^{-6}} = 20\,mA$$

$$\Rightarrow \quad 2\,mA$$

$$R_A = \frac{SG}{S+G}$$

$$R_A = \frac{20\times10^{-3}\times10}{10} = 20\times10^{-3}\,\Omega$$

$$i = \frac{\varepsilon}{\left(\frac{10^3\times R_V}{10^3 + R_V} + R_A\right)} = \frac{\varepsilon}{\left(\frac{1000\times50\times10^3}{51\times10^3}\right)}$$

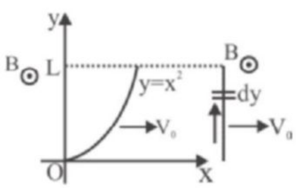
$$= \frac{51\varepsilon}{5\times10^4} \qquad (\because R_A \to 0)$$

$$i' = i\left(\frac{R_V}{51\times10^3}\right) = \frac{\varepsilon}{1000}$$

Measured resistance

$$\therefore \; R_m = \frac{i'\times1000}{i} = \frac{\varepsilon}{51\varepsilon}\times5\times10^4 = \frac{5\times10^4}{51} = 980.4\,\Omega$$

3.Sol:



For calculating the motional emf across the length of the wire, let us project wire such that $\vec{B}, \vec{V}, \vec{l}$ becomes mutually orthogonal. Thus

$$d\varepsilon = BV_0\,dy = B_0\left[1 + \left(\frac{y}{L}\right)^{\beta}\right]V_0\,dy$$

$$\varepsilon = \int_0^L B_0\left(1 + \left(\frac{y}{L}\right)^{\beta}\right)V_0 dy$$

$$= B_0 V_0 L\left[1 + \frac{1}{\beta+1}\right]$$

emf in loop is proportional to L for given value of β.

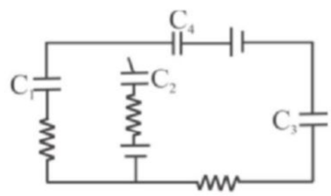
for

$$\beta = 0; \quad \varepsilon = 2B_0V_0L$$

$$\beta = 2; \quad \varepsilon = B_0V_0L\left[1 + \frac{1}{3}\right] = \frac{4}{3}B_0V_0L$$

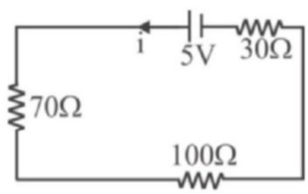
The length of the projection of the wire y = x of length $\sqrt{2}L$ on the y-axis is L thus the answer remain unchanged

4.Sol:



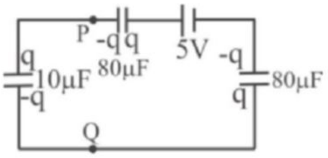
Just after closing of switch S_1 charge on capacitors is zero.

$\therefore$ Replace all capacitors with wire.



$$i = \frac{5}{70+100+30} = \frac{5}{200} = 25\text{mA}$$

Now S_1 is kept closed for long time and the circuit is in steady state. Current does not flow in the circuit.

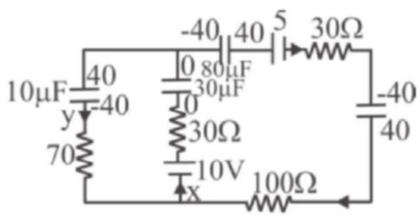


$$\frac{q}{10}+\frac{q}{80}+\frac{q}{80}-5=0$$

$$\frac{10q}{80}=5 \quad \Rightarrow \quad q=40\mu C$$

$\therefore$ V across $C_1 = 40/10 = 4V$

Now just after closing of S_2 charge on each capacitor remain same.



$$-10 + x\times 30 + 40/10 + y\times 70 = 0$$

$$30x + 70y = 6 \qquad (1)$$

$$-\frac{40}{80}+5+(x-y)30-\frac{40}{80}+(x-y)\times 100-10 +x\times 30=0$$

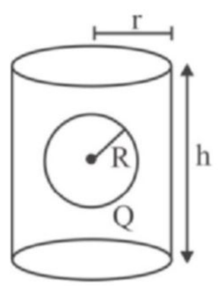
$$160x-130y-6=0 \qquad (2)$$

$$y = 96/1510$$

$$x = 0.05\,\text{amp}$$

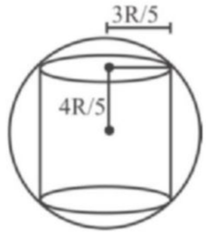
Correct option – 2, 3

5.Sol: If $h > 2R$ $\qquad r > R$



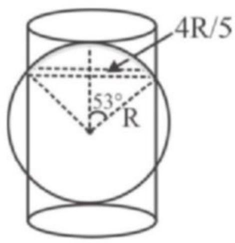
$\phi = \dfrac{Q}{\varepsilon_0}$ clearly from Gauss' Law

If $h=\dfrac{8R}{5}$ & $r=\dfrac{3R}{5}$



So for $h<\dfrac{8R}{5} \Rightarrow \phi = 0$

If for $h=2R$ & $r=\dfrac{4R}{5}$



$$Q_{enclosed} = 2\times 2\pi(1-\cos 53^\circ)\times\frac{Q}{4\pi}=\frac{2Q}{5}$$

$$\therefore \ \phi=\frac{2Q}{5\varepsilon_0}$$

$\therefore$ For $h>2R$ $r=\dfrac{4R}{5}$ $\quad \therefore \ \phi=\dfrac{2Q}{5\varepsilon_0}$

If $h=2R$ & $r=\dfrac{3R}{5}$

$$q_{enclosed} = 2\times 2\pi(1-\cos 37^o)\frac{Q}{4\pi}=\frac{Q}{5}$$

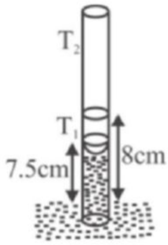
$$\therefore \ \phi=\frac{Q}{5\varepsilon_0}$$

6.Sol: When only single material tubes are used

$$h_1 = \frac{2T\cos\theta_1}{\rho r g}=\frac{2\times 0.075\times\cos 0^o}{1000\times 2\times 10^{-4}\times 10}=7.5\,\text{cm}$$

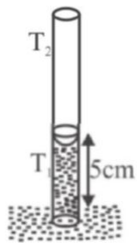
$$h_2 = \frac{2T\cos\theta_2}{\rho r g} = \frac{2\times 0.075\times \cos 60^o}{1000\times 2\times 10^{-4}\times 10} = 3.75\,\text{cm}$$

Case-1



Therefore, Option (a) is correct

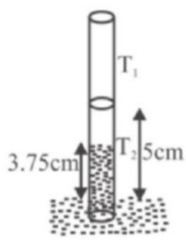
Case-1



(2) Liquid will rise only upto height of 5 cm and meniscus will adjust by changing its radius of curvature.
If the liquid goes up in tube 2 then it will not be able to support the weight of the liquid

(3) Weight of water in meniscus will be different in two cases because angle of contact is different .

(4) **Case-1I**



7.Sol: For $n_1 = n_2 = n$ $\quad \frac{1}{f} = \frac{2(n-1)}{R}$ (1)

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_2} = (n+\Delta n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f+\Delta f} = \frac{(n-1)}{R} + (n+\Delta n-1)\left(\frac{1}{R}\right)$$

$$\frac{1}{f+\Delta f} = \frac{2n+\Delta n-2}{R} \quad (2)$$

$$(1)/(2) \Rightarrow \frac{f+\Delta f}{f} = \frac{\frac{2(n-1)}{R}}{\frac{2n+\Delta n-2}{R}}$$

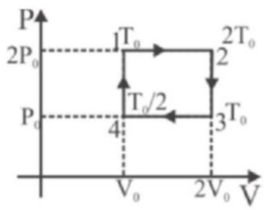
$$1 + \frac{\Delta f}{f} = \frac{2(n-1)}{2n+\Delta n-2}$$

$$\frac{\Delta f}{f} = \frac{-\Delta n}{(2n+\Delta n-2)}$$

$$\frac{\Delta f}{20} = -\frac{10^{-3}}{3+10^{-3}-2} \Rightarrow \Delta f_0 = -2\times 10^{-2}$$

$$|\Delta f_0| = 0.02\,\text{cm}$$

8.Sol:



$$\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{3\to 4}}\right| = \left|\frac{nC_P\Delta T_{1\to 2}}{nC_P\Delta T_{3\to 4}}\right| = \frac{T_0}{T_0/2} = 2$$

$$\left|\frac{\Delta Q_{1\to 2}}{\Delta Q_{2\to 3}}\right| = \left|\frac{nC_P\Delta T_{1\to 2}}{nC_V\Delta T_{2\to 3}}\right| = \frac{C_P}{C_V} = \frac{5}{3}$$

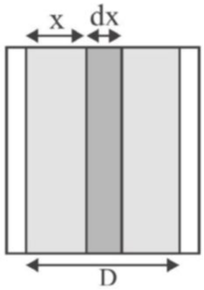
$$W_{cycle} = P_0V_0 = nR\left[\frac{T_0}{2}\right] = \frac{RT_0}{2}$$

option (c) is wrong as no adiabatic process is involved

Correct options are a,b

SECTION-3

1.Sol:



$$\frac{x}{m}=\frac{D}{N}$$

$$d\left(\frac{1}{C}\right)=\frac{dx}{K_m\varepsilon_0 A}=\frac{dx}{K\varepsilon_0 A\left(1+\frac{m}{N}\right)}$$

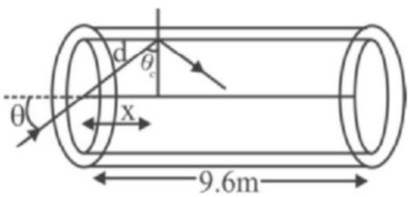
$$=\frac{dx}{K\varepsilon_0 A\left(1+\frac{x}{D}\right)}$$

$$\frac{1}{C_{eq}}=\int d\left(\frac{1}{C}\right)=\int_0^D\frac{Ddx}{K\varepsilon_0 A(D+x)}$$

$$\frac{1}{C_{eq}}=\frac{D}{K\varepsilon_0 A}\ln(2)$$

$C_{eq}=\frac{K\varepsilon_0 A}{D\ln 2}$. Therefore $\alpha=1$

2.Sol:



$$1.5\sin\theta_c=1.44\sin 90°$$

$$\sin\theta_C=\frac{1.44}{1.50}=\frac{24}{25}$$

$$\therefore\ \sin\theta_C=\frac{x}{d}=\frac{24}{25}$$

$$d=\frac{25x}{24}$$

$\therefore$ Total length traveled by light

$$=\frac{25}{24}\times 9.6=10m$$

$$\therefore\quad t=\frac{10}{\frac{3\times10^8}{1.5}}=5\times10^{-8}$$

$$t=50ns$$

3.Sol: Let S, L & W are the specific heat of liquid, latent heat of liquid and water equivalent of calorimeter respectively

Case-1 If calorimeter is open and after evaporation liquid escapes

$$5\times S\times(80-30)+5L=W\times30 \qquad (1)$$

$$80\times S\times20=W\times30$$

$$80\times S\times20=5\times S\times50+5L$$

$$5L=1350S$$

$$\frac{L}{S}=270$$

Case-2 If calorimeter is closed (vapour does not escape)

Heat gain = Heat loss

$$5S(80-30)+5L=W(110-80)$$

$$250S+5L=W\times30 \qquad (1)$$

Now 80gm liquid is poured

Heat gain = Heat loss

Here final temperature $=50^{o}C$

$$80\times S\times20=5L+5S\times30+W\times30 \qquad (2)$$

From (1) & (2)

$$\frac{L}{S}=120\text{Ans}$$

4.Sol: The small work done is $dW=\vec{F}.d\vec{r}$

$$dW=\alpha ydx+2\alpha xdy$$

$$A\to B\quad y=1,\ dy=0$$

$$\Rightarrow W_{A\to B}=\int\alpha ydx=\alpha\cdot1\int_0^1 dx=\alpha$$

$$B\to C\quad x=1,\ dx=0$$

$$\Rightarrow W_{B\to C}=2\alpha\cdot1\int_1^{0.5}dy=-2\alpha(0.5)=-\alpha$$

$C \rightarrow D \quad y = 0.5 \; dy = 0$

$$\Rightarrow W_{C\rightarrow D} = \int_1^{0.5} \alpha y dx = \alpha.\frac{1}{2}\int_1^{0.5} dx = -\frac{\alpha}{4}$$

$$\int_1^{0.5} dx = -\frac{\alpha}{4}$$

$E \rightarrow F, y = 0, dy = 0, \quad W_{EF} = 0$

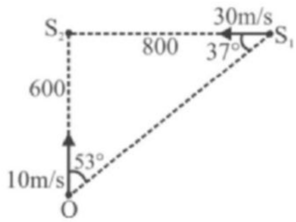
$F \rightarrow A \; x = 0, dx = 0, \quad W_{F\rightarrow A} = 0$

$$\therefore \quad W = \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4}$$

Given $\alpha = -1$

$\Rightarrow \quad W = 0.75 J$

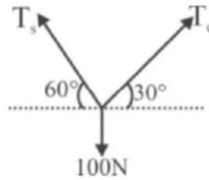
5.Sol:



$$f_b = 120\left[\left(\frac{330 + 10\cos 53°}{330 - 30\cos 37°}\right) - \left(\frac{330 + 10}{330}\right)\right]$$

$$= 120\left[\frac{336}{306} - \frac{34}{33}\right] = 8.128 Hz$$

6.Sol:



$$\frac{T_s}{2} = \frac{\sqrt{3}}{2} T_c$$

$$T_s = \sqrt{3} T_c$$

$$\frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right)\left(\frac{l_c}{l_s}\right)\left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{1}\right)$$

$$\left(\frac{2\times 10^{11}}{1\times 10^{11}}\right) = 2$$

Previous years JEE MAIN Questions

KINETIC THEORY OF GASES [ONLINE QUESTIONS]

1. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas? **[2017]**

(a) $\frac{1}{2}nRT$ (b) 0 (c) $\frac{3}{2}nRT$ (d) $\frac{5}{2}nRT$

2. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_V) is: **[2017]**

(a) 6 (b) $\frac{7}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{5}$

3. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is: **[2015]**

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

4. Using equipartition of energy, the specific heat (in $J\,kg^{-1}K^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium =27) **[2015]**

(a) 410 (b) 25 (c) 1850 (d) 925

5. A gas molecule of mass M at the surface of the Earth has kinetic energy equivalent to $0°C$. If it were to go up straight without colliding with any other molecules, how high it would rise? Assume that the height attained is much less than radius of the earth. (k_B is Boltzmann constant). **[2014]**

(a) 0 (b) $\frac{273k_B}{2Mg}$ (c) $\frac{546k_B}{3Mg}$ (d) $\frac{819k_B}{2Mg}$

6. At room temperature a diatomic gas is found to have an r.m.s speed of $1930ms^{-1}$. The gas is **[2014]**

(a) H_2 (b) Cl_2 (c) O_2 (d) F_2

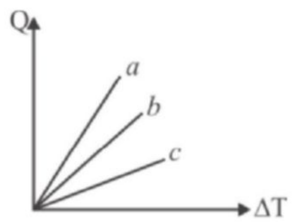
7. Modern vaccum pumps can evacuate a vessel down to a pressure of $4.0\times10^{-15}\,atm.$ At room temperat (300K). Taking $R=8.0JK^{-1}mole^{-1}$, and $N_{Avogadro}=6\times10^{23}\,mole^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of: **[2014]**

(a) $0.2\mu m$ (b) 0.2 mm
(c) 0.2 cm (d) 0.2 nm

8. In the isothermal expansion of 10g of gas from volume V to 2V the work done by the gas is 575J. What is the root mean square speed of the molecules of the gas at that temperature? **[2013]**

(a) 398 m/s (b) 520 m/s
(c) 499 m/s (d) 532m/s

9. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to monoatomic (M), diatomic (D) and polyatomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to: **[2013]**



(a) P, M and D (b) M, D and P
(c) P, D and M (d) D, M and P

[OFFLINE QUESTIONS]

1. The temperature of an open room of volume $30\,m^3$ increases from $17°C$ to $27°C$ due to sunshine. The atmospheric pressure in the room remains $1\times10^5\,Pa$. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be: **[2017]**

(a) 2.5×10^{25} (b) -2.5×10^{25}
(c) -1.61×10^{23} (d) 1.38×10^{23}

2. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b: **[2017]**

(a) a =14b (b) a = 28b
(c) $a = \frac{1}{14}b$ (d) a=b

3. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$ **[2015]**

(a) $\frac{3\gamma-5}{6}$ (b) $\frac{\gamma+1}{2}$ (c) $\frac{\gamma-1}{2}$ (d) $\frac{3\gamma+5}{6}$

4. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now ? (Atmospheric pressure = 76 cm of Hg) **[2014]**

(a) 38 cm (b) 6 cm
(c) 16 cm (d) 22 cm

5. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it's suddenly brought to rest. Assuming no heat is lost to the surrounding, its temperature increases by **[2011]**

(a) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$ (b) $\frac{\gamma M^2 v}{2R}K$

(c) $\frac{(\gamma-1)}{2R}Mv^2K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

6. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is: **[2011]**

(a) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

(b) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

(c) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

(d) $\frac{(T_1+T_2+T_3)}{3}$

7. One kg of a diatomic gas is at a pressure of $8\times10^4 N/m^2$. The density of gas is $4kg/m^3$. What is the energy of the gas due to its thermal motion? **[2009]**

(a) $5\times10^4 J$ (b) $6\times10^4 J$
(c) $7\times10^4 J$ (d) $3\times10^4 J$

8. The speed of sound in oxygen (O_2) at certain temperature is $460ms^{-1}$. The speed of sound in helium (*He*) at the same tempreature will be (assume both gases to be ideal) **[2008]**

(a) $1421ms^{-1}$ (b) $500ms^{-1}$
(c) $650ms^{-1}$ (d) $330ms^{-1}$

9. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[2007]**

(a) $C_P-C_V=28R$ (b) $C_P-C_V=R/28$
(c) $C_P-C_V=R/14$ (d) $C_P-C_V=R$

CALORIMETRY
[ONLINE QUESTIONS]

1. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto $150°C$. Immediately, it is put into water of volume 150 cc at $25°C$ kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is $40°C$. The specific heat of aluminium is: **[2017]**

(a) $378J/kg-°C$ (b) $315J/kg-°C$
(c) $476J/kg-°C$ (d) $499J/kg-°C$

2. An experiment takes 10 minutes to raise the temperature of water in a container from $0°C$ to $100°C$ and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be $1cal\ g-°C$, the heat of vapourisation according to this experiment will come out to be **[2015]**

(a) 560 cal/g (b) 550 cal/g
(c) 540 cal/g` (d) 530 cal/g

3. A kettle with 2 L water at $27°C$ is heated by operating coil heater of power 1kW. The heat is lost to the atmosphere at constant rate 160 J, when it is open. In how much time will water be heated to $77°C$ (sp. heat of water = 4.2 kJ/kg) with lid open? **[2014]**

(a) 14 min (b) 7 min
(c) 8 min 20 s (d) 6 min 2 s

4. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from $30°C$ to $25°C$. A mass of 100g of another liquid in an identical vessel with identical surroundings takes the same time to cool from $30°C$ to $25°C$. The specific heat of the liquid is: (The water equivalent of the vessel is 30g.) **[2013]**

(a) 2.0 kcal/kg (b) 7 kcal/kg
(c) 3 kcal/g (d) 0.5 kcal/kg

5. 500 g of water and 100g of ice at $0°C$ are in a calorimeter whose water equivalent 40g. 10g of steam at $100°C$ is added to it. then water in the calorimeter is: (Latent heat of ice $=80cal/g$, Latent heat of steam $=540cal/g$) **[2013]**

(a) 580 g (b) 590 g (c) 600 g (d) 610 g

[OFFLINE QUESTIONS]

1. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. subsequently, the temperature of the system is found to be $75°C$. T is given by (Given: room temperature $=30°C$, specific heat of copper $0.1cal/gm-°C$) **[2017]**

(a) $1250°C$ (b) $825°C$
(c) $800°C$ (d) $885°C$

2. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporisation. **[2013]**

(a) $\rho L / T$ (b) $\sqrt{T / \rho L}$

(c) $T / \rho L$ (d) $2T / \rho L$

3. 100 g of water is heated from $30°C$ to $50°C$. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg K) **[2011]**

(a) 8.4 kJ (b) 84 kJ (c) 2.1 kJ (d) 4.2 kJ

4. The specific heat capacity of a metal at low temperature (T) is given as

$$C_p\left(kHK^{-1}kg^{-1}\right) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from $20°K$ to $4°K$ by a special refrigerator operating at room temperature $(27°C)$. The amount of work required to cool the vessel is **[2011]**

(a) Greater than 0.148 kJ
(b) Between 0.148 kJ and 0.028 kJ
(c) Less than 0.028 kJ
(d) Equal to 0.002 kJ

ANSWER KEY

KINETIC THEORY OF GASES [ONLINE QUESTIONS]

1. a	2. d	3. c	4. d	5. d
6. a	7. b	8. c	9. c	

[OFFLINE QUESTIONS]

1. b	2. a	3. b	4. c	5. c
6. a	7. a	8. a	9. b	

CALORIMETRY [ONLINE QUESTIONS]

1. d	2. b	3. c	4. d	5. b

[OFFLINE QUESTIONS]

1. d	2. d	3. a	4. d

HINTS & SOLUTIONS

KINETIC THEORY OF GASES [ONLINE QUESTIONS]

1.Sol: Energy associated with N moles of diatomic gas, $U_i = N\frac{5}{2}RT$

Energy associated with n moles of mono atomic gas

$$= n\frac{3}{2}RT$$

Total energy when n moles of diatomic gas converted into monoatomic

$$(U_f) = 2n\frac{3}{2}RT + (N-n)\frac{5}{2}RT$$

$$= \frac{1}{2}nRT + \frac{5}{2}NRT$$

Change in total kinetic energy of the gas

$$\Delta U = \frac{1}{2}nRT$$

2.Sol: The ratio of specific heats at constant pressure (C_P) and constant volume (C_V)

$$\frac{C_P}{C_V} = \gamma = \left(1 + \frac{2}{F}\right)$$

where F is degree of freedom

$$\frac{C_P}{C_V} = \left(1 + \frac{2}{5}\right) = \frac{7}{5}$$

3.Sol: Pressure, $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

$$P \propto (v_{rms})^2 \propto T$$

So, force $\propto (v_{rms})^2 \propto T$

i.e., Value of q=1

4.Sol: According to Dulong-Petit law many solids at room temperature $C_V = 3R$. For solids when heat is given their volume approximately remain constant.

$$\Delta U = \Delta Q$$

For on mole

Using equipartition of energy, we have

$$3RT = MCT$$

$$C = \frac{3R}{M} = \frac{3\times 8.314}{27\times 10^{-3}}$$

$$\therefore C = 925 J/kgK$$

5.Sol: Kinetic energy of each molecule,

$$K.E = \frac{3}{2}K_B T$$

Given that $T = 273K$

Height attained by the gas molecule, h=?

$$K.E = \frac{3}{2}K_B(273) = \frac{819K_B}{2}$$

$$K.E = P.E$$

$$\Rightarrow \frac{819K_B}{2} = Mgh \Rightarrow h = \frac{819K_B}{2Mg}$$

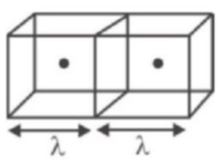
6.Sol: $\because c = \sqrt{\frac{3RT}{M}}$

$$(1930)^2 = \frac{3\times 8.314\times 300}{M}$$

$$M = \frac{3\times 8.314\times 300}{1930\times 1930} \approx 2g$$

The gas is H_2.

7.Sol: From the given information we calculate the average volume occupied by each molecule (not the volume of molecule). The average volume occupied or share of each molecule is equal to the ratio of total volume divides by total number of molecules. This volume can be approximated as a cube of side length (λ). The cube root of the average volume per molecule is equal to (λ). Here the value (λ) is also called as mean free path and it is equal to average distance between two molecules of a gas as shown in the figure.



Ideal gas equation is

$$PV = nRT = \frac{N}{N_A}RT$$

Given that

$$R = 8\frac{J}{K.mole}, T = 300K$$

$$N_A = 6\times 10^{23} mole^{-1}$$

$$P = 4.0\times 10^{-15}\times 10^5 \, N/m^2$$

$$\frac{V}{N} = \frac{RT}{PN_A} = \frac{8\times 300}{4.0\times 10^{-15}\times 10^5\times 6\times 10^{23}}$$

$$\lambda^3 = \frac{V}{N} = 10^{-11} m^3$$

$$\lambda = 2.15\times 10^{-4} = 0.215mm$$

8.Sol: In isothermal process temperature remains constant.

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{Mn}} = \sqrt{\frac{3PV}{m}} \qquad \text{(i)}$$

Isothermal work done is

$$W = nRT\ln\frac{V_2}{V_1} = PV\ln 2$$

Given that $W = 575J = PV\ln 2$

$$\Rightarrow \quad PV = \frac{575}{\ln(2)} \qquad \text{(ii)}$$

From eq's (i) & (Ii) we get

$$v_{rms} = \sqrt{\frac{3\times 575}{m(\ln 2)}} = \sqrt{\frac{3\times 575}{10^{-2}\times 0.693}} = 499m/s$$

9.Sol: In isobaric process the heat given to a gas is

$$Q = nC_P\Delta T$$

The graph between Q and ΔT is a straight line passing through origin.

$\text{slope} = nC_P$

As n same for all the gases

We know that

$$C_{P(\text{Poly})} > C_{P(\text{Di})} > C_{P(\text{Mono})}$$

a,b,c correspond to P, D and M

[OFFLINE QUESTIONS]

1.Sol: Given: $T_i = 17 + 273 = 290K$

$T_f = 27 + 273 = 300K$

Atmospheric pressure, $P_0 = 1\times10^5\, Pa$

Volume of room, $V_0 = 30m^3$

$$N = \frac{PVN_A}{RT}$$

Given that number of molecules $N=n$

$$\therefore n_f - n_i = \frac{P_0V_0}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)N_A$$

$$= \frac{1\times10^5\times30}{8.314}\times6.023\times10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5\times10^{25}$$

2.Sol: Molar specific heat $C' = \frac{\Delta Q}{n\Delta T}$

Specific heat $C = \frac{\Delta Q}{m\Delta T}$

$$\frac{C'}{C} = \frac{m}{n} = M\,(\text{Molecular weight})$$

$$C = \frac{C'}{M}$$

$$C_P' - C_V' = R$$

$$MC_P - MC_V = R$$

$$C_P - C_V = \frac{R}{M}$$

For hydrogen M=2

$$C_P - C_V = \frac{R}{2} = a$$

For nitrogen M=28

$$\Rightarrow \quad C_P - C_V = \frac{R}{28} = b$$

$$\frac{a}{b} = 14$$

3.Sol: Average time between collisions

$$= \frac{\text{Mean free path}}{v_{\text{rms}}}$$

$$t = \frac{\dfrac{1}{\pi d^2 N/V}}{\sqrt{\dfrac{3RT}{M}}};$$

$$t = \frac{CV}{\sqrt{T}} \quad \left(\text{where } C = \frac{\sqrt{M}}{\pi d^2 N\sqrt{3R}}\right)$$

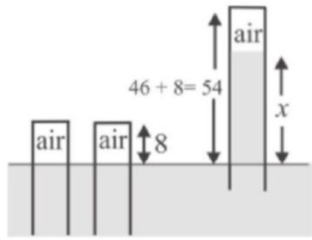
$$\Rightarrow T \propto \frac{V^2}{t^2}$$

Given that $TV^{\gamma-1} = k$

$$\frac{V^2}{t^2}V^{\gamma-1} = k \Rightarrow \frac{V^{\gamma+1}}{t^2} = k$$

$$t \propto V^{\frac{\gamma+1}{2}} \Rightarrow q = \frac{\gamma+1}{2}$$

4.Sol: The pressure inside the tube after closing the tube is 76 *cm* of mercury.



We assume that T is constant

$$P_1V_1 = P_2V_2$$

After pulling the tube up the pressure of the air is 76 *cm* - x, where x is the rise in the level of mercury

$$(76)(8) = (54 - x)(76 - x)$$

$$\Rightarrow x^2 - 130x + 3496 = 0$$

$$x = 38\ cm$$

Length of air column= 54- 38 = 16 cm

5.Sol: Here, work done is zero.

So, loss in kinetic energy = Change in internal energy
of gas

$$\frac{1}{2}mv^2 = nC_V\Delta T = n\frac{R}{\gamma-1}\Delta T$$

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$$

$$\therefore \ \Delta T = \frac{Mv^2(\gamma - 1)}{2R} K \quad (K - Kelvin)$$

6.Sol: Number of moles of first gas $= \frac{n_1}{N_A}$

Number of of moles of second gas $= \frac{n_2}{N_A}$

Number of of moles of third gas $= \frac{n_3}{N_A}$

By conserving total internal energy

$$U_f = U_i$$

$$\frac{n_1}{N_A} RT_1 + \frac{n_2}{N_A} RT_2 + \frac{n_3}{N_A} RT_3 = \frac{n_1 + n_2 + n_3}{N_A} RT_{mix}$$

$$T_{mix} = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$$

7.Sol: $K.E = \frac{5}{2} nRT = \frac{5}{2} PV$

$$= \frac{5}{2} \frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4 J \quad \left(V = \frac{m}{d}\right)$$

8.Sol: The speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \frac{v_{O_2}}{v_{He}} = \sqrt{\frac{\gamma_{O_2}}{M_{O_2}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{1.4}{32} \times \frac{4}{1.67}} = 0.3237$$

$$\therefore v_{He} = \frac{v_{O_2}}{0.3237} = \frac{460}{0.3237} = 1421 m/s$$

9.Sol: Molar specific heat $C' = \frac{\Delta Q}{n\Delta T}$

Specific heat $C = \frac{\Delta Q}{m\Delta T}$

$$\frac{C'}{C} = \frac{m}{n} = M \text{(Molecular weight)}$$

$$C = \frac{C'}{M}$$

$$C'_P - C^1_V = R$$

$$MC_P - MC_V = R \Rightarrow \quad C_P - C_V = \frac{R}{M}$$

For nitrogen M=28

$$C_P - C_V = \frac{R}{28}$$

CALORIMETRY [ONLINE QUESTIONS]

1.Sol: According to principle of calorimetry,

$$Q_{\text{given}} = Q_{\text{used}}$$

$$0.2 \times s \times (150 - 40)$$

$$= 0.15 \times 1(40 - 25) + 0.025(40 - 25)$$

$$s = \frac{0.175(15)}{0.2 \times 110} = 0.1193 \frac{cal}{g°C} = 499.46 \frac{J}{kg°C}$$

2.Sol: As $Pt = mc\Delta T$

$$P \times 10 \times 60 = mc \times 1 \times 100$$

(i) and $P \times 55 \times 60 = mL$

(ii) Dividing equation (i) by (ii) we get

$$\frac{10}{55} = \frac{100}{L}$$

$$\therefore L = 550 cal./g.$$

3.Sol: Energy given by heater must be equal to the sum of energy gained by water and energy lost from the lid.

$$Pt = ms\Delta T + \text{energy lost}$$

$$1000t = 2 \times (4.2 \times 10^3) \times 50 + 160t$$

$$840t = 8.4 \times 10^3 \times 50$$

⇒ t = 500 s = 8 min 20 s.

4.Sol: As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from $30°C$ to $25°C$) is same 2 minutes, therefore

$$\left(\frac{dQ}{dt}\right)_{\text{water}} = \left(\frac{dQ}{dt}\right)_{\text{liquid}}$$

$$\frac{(m_w c_w + W)\Delta T}{t} = \frac{(m_l c_l + W)\Delta T}{t}$$

(W= water equivalent of the vessel)

or, $m_w c_w = m_l c_l$

$\therefore$ Specific heat of liquid, $c_l = \frac{m_W c_W}{m_l}$

$$= \frac{50 \times 1}{100} = 0.5 \ kcal/kg$$

5.Sol: The heat released by steam when it converts from $100°C$ to $0°C$.

$$\Delta Q = 10\times 540 + 10\times 1\times 100$$

The mass of the ice that melts is

$$\Delta Q = 6400 = m80 \Rightarrow m = 80g$$

Water in calorimeter $= 500 + 80 + 10 = 590g$

[OFFLINE QUESTIONS]

1.Sol: According to principle of calorimetry,

Heat lost = Heat gain

$$100\times 0.1(T-75) = 100\times 0.1\times 45 + 170\times 1\times 45$$

$$10(T-75) = 450 + 7650$$

$$T = 885°C$$

2.Sol: Assume that radius is decreased by ΔR

$$\Delta mL = \Delta AT$$

where Δm is the decrease in mass of the drop. ΔA is decrease in surface area. T is the coefficient of surface tension.

$$4\pi R^2 \Delta R \rho L = 4\pi T\left[R^2 - (R-\Delta R)^2\right]$$

$$\Rightarrow \rho R^2 \Delta RL = T\left[R^2 - R^2 + 2R\Delta R - \Delta R^2\right]$$

$$\Rightarrow \rho R^2 \Delta RL = T2R\Delta R \quad [\Delta R \text{ is very small}]$$

$$\Rightarrow R = \frac{2T}{\rho L}$$

3.Sol: $\Delta U = \Delta Q = mc\Delta T$

$$= 100\times 10^{-3}\times 4184(50-30) \approx 8.4kJ$$

4.Sol: Required work = Energy released

Here, $Q = \int mc\,dT$

$$= \int_{20}^{4} 0.1\times 32\times\left(\frac{T^3}{400^3}\right)dT \approx 0.002kJ$$

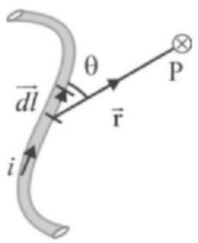
Therefore, required work = 0.002 kJ.

QUICK RECAP

For competitive edge

21. Magnetic Effects of Current

1.Biot - Savart's Law



The field produced by the small element at P is

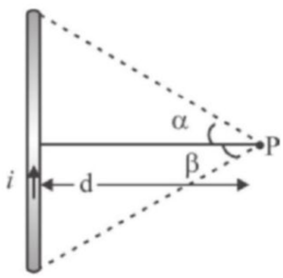
$$d\bar{B} = \frac{\mu_o}{4\pi}\frac{id\bar{l}\times\bar{r}}{r^3}$$

Here $\frac{\mu_0}{4\pi}$ is the proportionality constant.

Where μ_o is the permeability of the medium

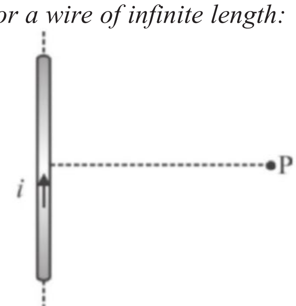
$$\frac{\mu_o}{4\pi} = 10^{-7}\,Hm^{-1}$$

2. Magnetic field Due to a straight Wire



$$B = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)$$

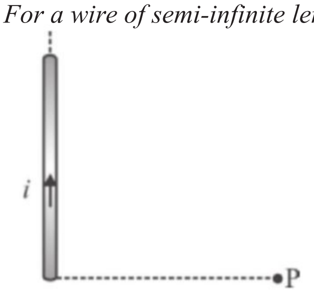
(I) For a wire of infinite length:



$$\alpha = \beta = 90^o$$

So, $B = \frac{\mu_0 i}{4\pi d}[\sin 90^o + \sin 90^o] = \frac{\mu_o i}{2\pi d}$

(II) For a wire of semi-infinite length:



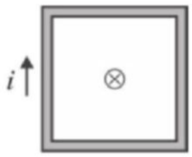
$\alpha = 90^o$ and $\beta = 0^o$

So, $B = \frac{\mu_0 i}{4\pi d}[\sin 90^o + \sin 0^o]$

$$B=\frac{\mu_0 i}{4\pi d}$$

(III) For axial position of wire:

- Magnetic induction at the centre of current carrying wire bent in the form of square of side 'a' is



$$B=8\sqrt{2}\left(\frac{\mu_0 i}{4\pi a}\right)$$

3. $\vec{B}$ due to current carrying circular Coil

Magnetic Induction on the axis of a circular current carrying coil is :

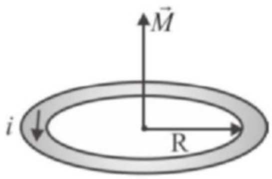
$$B=\frac{\mu_0 niR^2}{2\left(R^2+x^2\right)^{\frac{3}{2}}}$$

Where n is the number of turns, 'R' is the radius of the coil and 'x' is the distance of the point from the centre of the coil.

Case-1: At the center of coil (x=0) $B=\frac{\mu_0 ni}{2R}$

Case-2 : If x>> R then $B=\frac{\mu_o niR^2}{2x^3}$

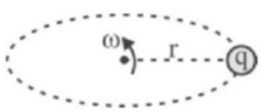
4. Magnetic Moment of a current loop



$$\vec{\mu}=i\pi R^2\hat{n}$$

Where $\hat{n}$ is Perpendicular to the plane of the loop

Magnetic moment of a rotating point charge:

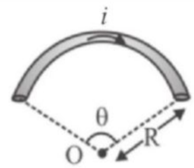


$$\mu=iA=\frac{q\omega}{2\pi}(\pi r^2)=\frac{q\omega r^2}{2} \quad \text{(i)}$$

5. Relation between L and μ

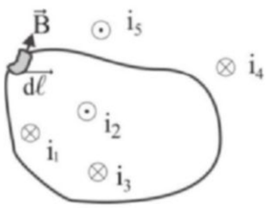
$$\mu=\left(\frac{q}{2m}\right)L$$

Magnetic Field by different current elements

Figure	Magnetic field
 Arc	$B=\frac{\mu_0}{4\pi}\cdot\frac{\theta i}{R}$
 Circular wire	$B=\frac{\mu_0}{4\pi}\cdot\frac{2\pi i}{R}=\frac{\mu_0 i}{2R}$

6. Ampere's Law (Circuital Law)

The line integral of $\vec{B}.d\bar{l}$ around any closed path is equal to μ_0 times the net current across the area bounded by this path



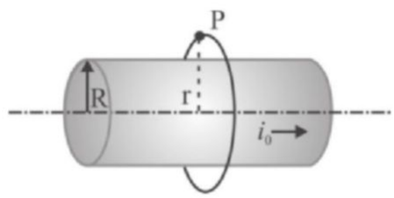
$$\oint \vec{B}.\,d\bar{l}=\mu_0\sum i$$

with respect to the above fig.

$$\sum i=i_1-i_2+i_3$$

7. $\vec{B}$ due to solid cylindrical conduting wire

Consider a cylindrical wire carrying current i_0



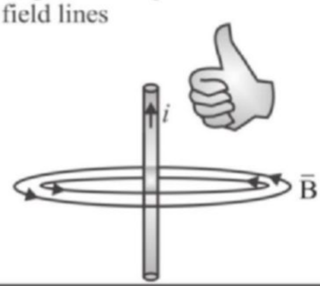
$$B = \frac{\mu_0 J}{2} r; \qquad \left(r \le R,\ J = \frac{i_0}{\pi R^2} \right)$$

$$\Rightarrow B = \frac{\mu_0 i_0}{2\pi r}; \qquad r \ge R$$

Right hand rule to find directions of $\bar{B}$ & $\bar{\mu}$

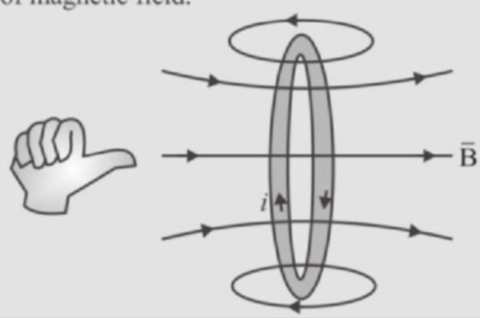
(i) Right hand thumb rule for straight current wire

Place the right hand thumb along the direction of current flow,then the direction of floding fingers will represent the direction of magnetic field lines



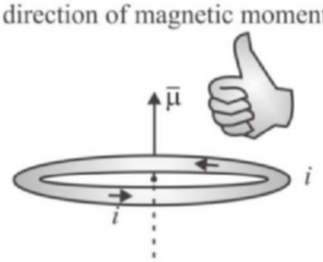
(ii) Right hand thumb rule for circular current wire

Fold the right hand four fingers in the direction of current, then the thumb indicates the direction of magnetic field.

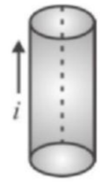


(iii) Right hand thumb rule for magnetic moment of a circular coil

Fold the right hand four fingers in the direction of current, then the direction of thumb indicates the direction of magnetic moment vector.



8. Thin hollow cylinder



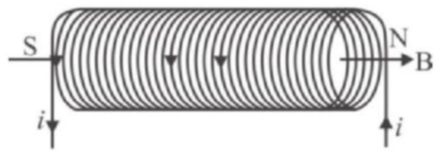
$r < R \qquad B = 0$

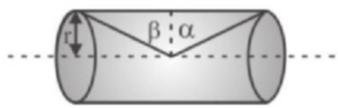
$r > R \qquad B = \dfrac{\mu_o i}{2\pi r}$

9. Magnetic field due to a Solenoid

- Field at a point on the axis of a solenoid is

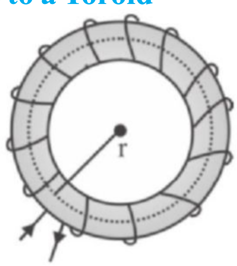
$$B = \frac{\mu_0 n i}{2}(\sin\alpha + \sin\beta)$$





- For a long solenoid magnetic induction on the axis is $B = \mu_0 n i \qquad (\because \alpha = \beta = 90^0)$
- At one end of a long solenoid magnetic induction is $B = \dfrac{\mu_0 n i}{2} \quad (\because \alpha = 0^0, \beta \cong 90^0)$
- Magnetic field outside the solenoid is approximately zero.

10. $\vec{B}$ due to a Toroid



$B = \dfrac{\mu_0 N i}{2\pi r}$ (r >> diameter of the core of the toroid)

11. Force on a Charge Moving in $\vec{B}$

- When a charged particle having charge q travels with velocity $\vec{v}$ in magnetic field $\vec{B}$ it experiences a force $\vec{F}$ given by $\vec{F} = q(\vec{v} \times \vec{B})$

12. Motion of a charged particle in magnetic field

(I) When $\vec{v}$ is perpendicular to $\vec{B}$:

$$\text{Radius} = \frac{mv}{qB} \quad (\text{since, } qvB = \frac{mv^2}{r})$$

$$\text{Frequency 'n'} = \frac{qB}{2\pi m}$$

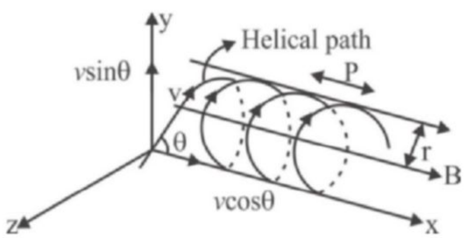
$$\text{Time period 'T'} = \frac{2\pi m}{qB}$$

(II) When the angle between $\vec{v}$ and $\vec{B}$ is θ (other than 0^0, 90^0 (or) 180^0)

$$r = \frac{m(v\sin\theta)}{qB}$$

$$\text{Time period} = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$$

$$x = T(v\cos\theta) = \frac{2\pi m(v\cos\theta)}{qB}$$



13. Force on a charge moving in $\bar{\mathbf{B}}$ & $\bar{\mathbf{E}}$

$\vec{F} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$, which is the famous **'Lorentz-force equation'**.

14. Cyclotron

- The cyclotron is a device used to accelerate charged particles or ions to high energy.
- The period of revolution of charges is given by

$$T = \frac{1}{f_c} = \frac{2\pi m}{Bq}$$

Here f_c is called the cyclotron frequency.

$$v = \frac{BqR}{m}$$

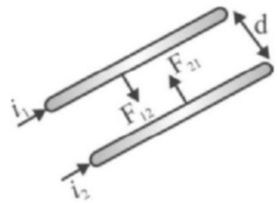
15. Magnetic Force on a current wire

$$\vec{F} = \int d\vec{F} = i\int (d\vec{l} \times \vec{B}).$$

If magnetic field is uniform, i.e., $\vec{B}$ = constant,

$$\vec{F} = i[\int d\vec{l}] \times \vec{B} = i(\vec{l} \times \vec{B})$$

(1) Force between two long current wires



- Force experienced per unit length of each conductor is,

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi r}$$

where i_1, i_2, are the currents flowing through the two conductors.

r is the perpendicular distance between them.

16. Torque on a Current Carrying Coil

- If the area vector of a coil makes an angle 'θ' with the direction of the uniform field of induction B then

$$\tau = nIAB\sin\theta$$

$$\vec{\tau} = \vec{M} \times \vec{B}$$

Where 'A' is area of the coil of 'n' turns carrying a current I and magnetic moment of coil $M = nIA$.

17. P. E of a coil in uniform magnetic field

- If the angle made by $\overline{M}$ of the coil with $\overline{B}$ in uniform magnetic field is 'θ', then its potential energy $U = -\overline{M}.\overline{B}$ $\quad U = -MB\cos\theta$

S.No	Situation	Formula
1.	Magnetic field at a point on the axis at distance x from the centre of a current carrying circular loop	$\frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$
2.	Magnetic field at the centre of a current carrying circular loop	$\frac{\mu_0 i}{2R}$
3.	Magnetic field on the axis of a current carrying circular loop far away from the centre of the loop	$\frac{\mu_0}{4\pi}\frac{2M}{x^3}$ (Loop behaves as magnetic dipole)
4.	Magnetic field on the centre of current carrying circular arc	$\frac{\mu_0}{4\pi}\frac{i}{r}\theta$
5.	Magnetic field due to a long thin current carrying wire	$B=\frac{\mu_0 i}{2\pi r}$
6.	Magnetic field inside a long straight current carrying cylindrical conductor at a distance *r* from the axis	$B=\frac{\mu_0}{2\pi r}\cdot\frac{i}{R^2}\cdot r$
7.	Magnetic field inside a long solenoid	$B=\mu_o ni$
8.	Magnetic field inside a toroid	$B=\frac{\mu_0 Ni}{2\pi r}$

22. Magnetism & Matter

1. Magnet

An object that attracts iron, cobalt, nickel and some other materials.

2. Types Of Magnet

(1) Natural magnets:

The magnet which is found in nature is called a natural magnet.

Eg: Magnetite (Fe_2O_3).

(II) Artificial magnets:

The magnets which are artificially prepared are known as artificial magnets.

Eg: These are generally made of iron, steel and nickel.

3. Magnetic Length

The shortest distance joining the poles of the magnet is known as 'magnetic length'. It is denoted by $2l$ (some time with l).

- The magnetic length is 5 / 6 times the geometric length of a bar magnet.

4. Pole Strength(m)

The ability of a pole to attract or repel another pole of a magnet is called pole strength.

- S.I. unit of pole strength is ampere meter or weber.
- 1 weber = μ_0 Am

Where μ_0 is permeability of free space

(The ability of a medium to allow magnetic lines to pass through it is called permeability)

$$\mu_o = 4\pi \times 10^{-7} \text{ weber / A m}$$

5. Magnetic Moment $\vec{M}$:

$$M = m \times 2\ell$$

- Its unit is ampere metre^2.

6. Magnetic Field

- It is also called as magentic flux density (or) magnetic field vector $(\overline{B})$.
- Magnetic induction at a point in a magnetic field is defined as force experienced by a unit north pole placed at that point.

$$\vec{B} = \frac{\vec{F}}{m}$$

- S.I unit is tesla or weber / m^2
- The magnetic field induction due to a pole of pole strength m at a distance 'd' is given by $B_0 = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{d^2}$

7. Torque on a Magnetic Dipole

- In vector notation $\vec{\tau} = \vec{M} \times \vec{B}$

8. Potential energy of a magnetic dipole

$$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$$

- The work done by an external agent in deflecting a magnet from angular position θ_1 to an angular position θ_2 with the field is given as

$$W = \Delta U = U_2 - U_1 = MB(\cos\theta_1 - \cos\theta_2)$$

9. Field of a Bar Magnet

(1) Axial line: $B_a = \left(\frac{\mu_0}{4\pi}\right)\frac{2Md}{(d^2 - l^2)^2}$

For a short bar magnet i.e., $l^2 << d^2$;

then $B_a = \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{d^3}$

(2) Equatorial line:

$$B_e = \frac{\mu_0}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}$$

For a short bar magnet i.e., $l^2 << d^2$; then

$$B_e = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{d^3}$$

Comparison between Electric & Magnetic properties		
Physical Quantity	**Electric Property**	**Magnetic Property**
Charge	q (+ve charge)	m (strength of a north pole)
Coulomb's constant	$\frac{1}{4\pi\varepsilon_0}$	$\frac{\mu_0}{4\pi}$
Field	$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$	$B=\frac{\mu_0}{4\pi}\frac{m}{r^2}$
Potential	$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$	$V=\frac{\mu_0}{4\pi}\frac{m}{r}$
Force	$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$	$F=\frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2}$
Relation between force & field	$\overline{F}=q\overline{E}$	$\overline{F}=m\overline{B}$
Potential energy	$U=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$	$U=\frac{\mu_0}{4\pi}\frac{m_1m_2}{r}$
Relation between W & ΔU	$W_e=-\Delta U$	$W_B=-\Delta U$
Dipole moment	$p=qd$	$M=md$
Relation between field and potential	$E=-\frac{dV}{dr}$	$B=-\frac{dV}{dr}$
Potential difference	$V_2-V_1=-\int_{r_1}^{r_2}\overline{E}.d\overline{r}$	$V_2-V_1=-\int_{r_1}^{r_2}\overline{B}.d\overline{r}$
Relation between F and U(or) Force on a dipole	$F=-\frac{\partial U}{\partial r}$	$F=-\frac{\partial U}{\partial r}$
Potential by a short dipole	$\frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}$	$\frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$
Field on the axial line by a short dipole	$\frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$	$\frac{\mu_0}{4\pi}\frac{2M}{r^3}$
Field on the equatorial line by a short dipole	$\frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}$	$\frac{\mu_0}{4\pi}\frac{M}{r^3}$
Torque	$\overline{\tau}=\overline{P}\times\overline{E}$	$\overline{\tau}=\overline{M}\times\overline{B}$
Potential energy of a short dipole	$U=-\overline{P}.\overline{E}$	$U=-\overline{M}.\overline{B}$

At any point in the plane of axial and equatorial lines:

For a short magnet,

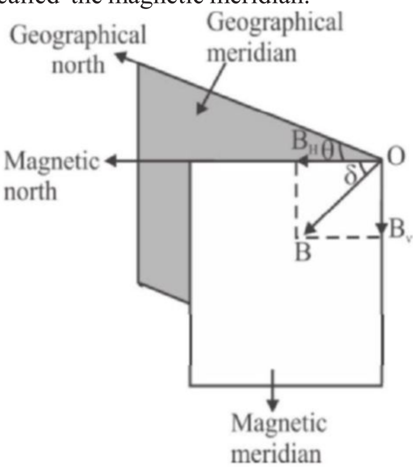
$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{\sqrt{(3\cos^2\theta + 1)}M}{d^3}$$

$\theta = 0^0$ for axial line ; $\theta = 90^0$ for equatorial line

10. Earth's Magnetic Field

(I) Geographical Meridian: A vertical plane passing through the axis of rotation of the earth is called the geographical meridian.

(II) Magnetic Meridian: A vertical plane passing through the axis of a freely suspended magnet is called the magnetic meridian.



$$\frac{B_V}{B_H} = \frac{B\sin\delta}{B\cos\delta} = \tan\delta$$

Where δ =dip (or) inclination; θ = declination

(III)Apparent dip

Relation between true dip (δ) and apprent dips $(\delta_1 \,\&\, \delta_2)$

$$\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$$

11. Moving-Coil Galvanometer(MCG)

The deflection torque is $\tau = NiAB$.

$$NiAB = k\theta \Rightarrow i = \frac{k}{NAB}\theta$$

$\frac{\theta}{i}$ - current sensitivity of the galvanometer

$\frac{\theta}{V}$ - voltage sensitivity of the galvanometer

(V = iR, R - is the resistance of the coil)

12. Vibration Magnetometer

Vibration magnetometer is used to find the horizontal component of earth magnetic field at a point. Time period of oscillation and frequency of magnet is

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \text{ and } n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$$

13. Tangent Law

If we have two uniform magnetic fields $\overline{B}$ and B_H at right angle to each other and if a bar magnet is placed in such a combination of fields.

The magnet will be acted upon by two couples. It will set in direction θ, such that the couples balance each other.

Deflection couple due to $\vec{B}$

$$= mB \times NO = mB.2l\cos\theta$$

$$= (2ml)B\cos\theta = MB\cos\theta$$

Resorting couple due to B_H

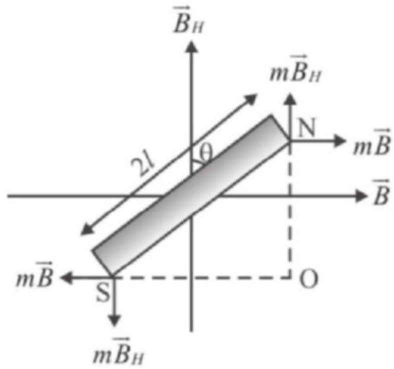
$$= mB_H \times SO = mB_H.2l\sin\theta$$

$$= (2ml)B_H\sin\theta = MB_H\sin\theta$$

$\therefore$ When the magnet is in the equillibrium position,

$$B = MB\cos\theta = MB_H\sin\theta$$

$$\Rightarrow B = B_H\tan\theta$$



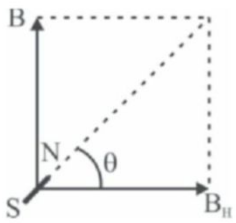
$\tan\theta = B / B_H$ (tangent law)

14. Tangent Galvanometer (TG)

- It is a moving magnet galvanometer used to measure current.

The magnetic field at the centre is

$$B = \frac{\mu_0 ni}{2r}$$



$$\tan\theta = \frac{B}{B_H} \qquad \Rightarrow B_H \tan\theta = \frac{\mu_0 n i}{2r}$$

- $i = \left(\frac{2r B_H}{\mu_0 n}\right)\tan\theta = K\tan\theta$

$\Rightarrow i = K\tan\theta$
Where K is called as T.G. constant (or) reduction factor of T.G., r-radius of the coil in meter;

B_H = Horizontal component of earth's magnetic field in tesla;

i = current in amperes

15. Deflection Magnetometer

(I)Tan A position : In this position the magnetometer is set perpendicular to magnetic meridian so that, magnetic field due to magnet, is in axial position and perpendicular to earth's field.

$$\Rightarrow B_H \tan\theta = \frac{\mu_0}{4\pi}.\frac{2Mr}{(r^2 - l^2)^2}$$

$$\Rightarrow B_H \tan\theta = \frac{\mu_0}{4\pi}.\frac{2M}{r^3} \quad (r>>l)$$

(II)Tan B position :The arms of magnetometer are set in magnetic meridian, so that the magnetic field due to magnet is at its equatorial position,

$$\Rightarrow B_H \tan\theta = \frac{\mu_0}{4\pi}.\frac{M}{(r^2 + l^2)^{3/2}}$$

$$\Rightarrow B_H \tan\theta = \frac{\mu_0}{4\pi}.\frac{M}{r^3} \quad (r>>l)$$

(III) Comparison of magnetic moments :

According to deflection method $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$

According to null deflection method

$$\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$$

16. Magnetic Field Intensity $(\vec{H})$

- The force experienced by unit north pole of one weber is called magnetic field strength or intensity of magnetic field strength. i.e., H = F / m
- SI unit : Am^{-1} or weber

17. Intensity of Magnetisation(I)

$$I = \frac{M}{V} = \frac{m(2l)}{A(2l)} = \frac{m}{A}$$

Where m = pole strength; M= Magnetic moment
a = area of cross-section of the magnet;
V=Volume of the magnet

- SI unit : Am^{-1}

18. Magnetic Susceptibility (χ)

$$\chi = \frac{I}{H}$$

It has no units

19. Magnetic permeability (μ)

i.e., $\mu = \frac{B}{H}$

- It is a scalar having unit Hm^{-1}

20. Relative permeability (μ_r)

$$\mu_r = \frac{\mu}{\mu_0}$$

- It has no units and dimensions

21. Relation between μ_r and χ

As $\mu_r = \frac{\mu}{\mu_0} \Rightarrow \mu_r = (1+\chi)$

22. Curie's Law:

It states that intensity of magnetisation (I) and hence χ are inversely proportional to the absolute temperature (T)

i.e., $\chi = \frac{C}{T}$ where C- Curie constant

23. Curie-Weiss Law:

At temperature above Curie temperaure the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_C)$

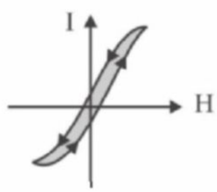
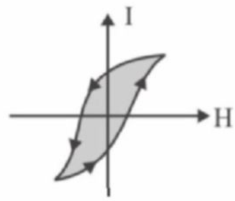
i.e., Susceptibility $\chi_m \alpha \frac{1}{T - T_C}$

Where T_C is Curie's temperature

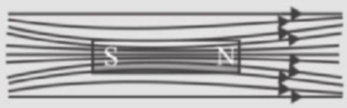
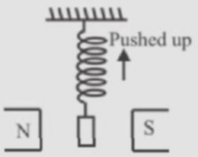
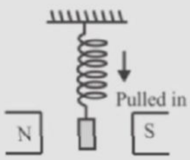
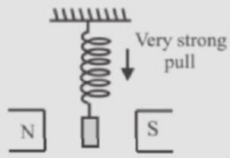
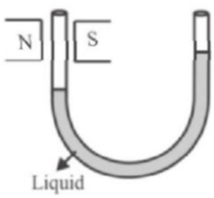
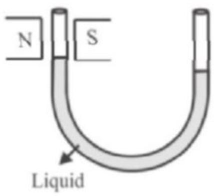
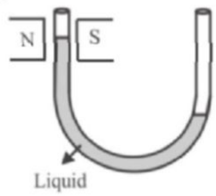
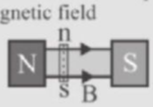
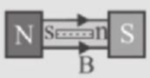
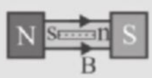
24. Hysteresis:

The phenomenon in which the value of a physical property lags behind changes in the effect causing it.

Comparison between soft iron and steel

Soft iron	Steel
 	
The area of hysteresis loop is less (low energy loss)	The area of hysteresis loop is large (high energy loss)
Retentivity and coercive force are small	Retentivity and coercive force are large
I and χ both are high	I and χ both are low
Magnetic permeability is high	Magnetic permeability is less
It can be magnetised and demagnetised easily	Magnetisation and demagnetisation is not easy
Used in transformer, dynamo and electromagnet tape recorder etc.	Used for making permanent magnet.

Comparative study of magnetic materials

Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
Diamagnetic substances are those substances which are feebly repelled by a magnet. Eg: Au, Cu, Ag, Zn, Sb, NaCl, air and diamond etc.	Paramagnetic substances are those substances which are feebly attracted by a magnet. Eg: Mn, Al, Pt, Na and crown glass	Ferromagnetic substances are those substances which are strongly attracted by a magnet. Eg: Fe, Co, Ni, Cd, Fe_2O_3 etc.
When placed in magnetic field, the lines of force tend to avoid the substance. 	The lines of force prefer to pass through the substance rather than air. 	The lines of force tend to crowd into the specimen 
Magnetism arises because of orbital motion of electrons	Magnetism arises because of Spin and orbital motion of electrons	Magnetism arises because of formation of domains
When a diamagnetic substance is placed in external magnetic field then it will be repelled. 	When a paramagnetic substance is placed in external magnetic field then it will be attracted. 	When a ferromagnetic substance is placed in external magnetic field then it will be strongly repelled. 
A U-tube having diamagnetic liquid is placed in between the poles of a magnet then the liquid level in the limb gets depressed 	A U-tube having paramagnetic liquid is placed in between the poles of a magnet then the liquid level in the limb rises up 	A U-tube having ferromagnetic liquid is placed in between the poles of a magnet then the liquid level in the limb rises up greatly. 
These are weakly magnetised in a direction opposite to that of applied magnetic field	These get weakly magnetised in the direction of applied magnetic field	These gets strongly magnetised in the direction of applied magnetic field
Intensity of Magnetisation(I) has a small -ve value.	Intensity of Magnetisation(I) has a small +ve value.	Intensity of Magnetisation(I) has a large +ve value.
Low and negative $\|\chi\|\approx1$	Low but positive $\|\chi\|\approx1$	Positive and high $\|\chi\|\approx10^{2}$
Relative permeability $\mu_r<1$	Relative permeability $\mu_r>1$	Relative permeability $\mu_r>>1$
If a rod of diamagnetic material is suspended freely between two magnetic poles, its axis becomes perpendicular to the magnetic field 	Paramagnetic rod becomes parallel to the magnetic field. 	Ferromagnetic rod also becomes parallel to the magnetic field. 

23. Electro Magnetic Induction

Electromagnetic Induction

1.Magnetic Flux(ϕ)

- The magnetic flux through a small surface of area dA is $d\phi = \vec{B}\cdot d\vec{A}$

  $\vec{B}$ -magnetic field,

  $d\vec{A}$ - area vector defined normal to the surface.
- Flux through a large surface is

$$\phi = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} \text{ (if } \vec{B} \text{ is uniform)}$$

SI units: Tesla/meter^2 or weber(Wb)

2. Faraday's laws of electromagnetic induction

(I) Induced E.M.F

$$e = -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt}$$

(II) Induced Current

$$I = \frac{e}{R} = -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

(III) Induced Charge

$$dq = i_{ind}dt = -\frac{1}{R}\left(\frac{d\phi}{dt}\right)dt = -\frac{d\phi}{R}$$

$$\Delta q = \frac{\Delta\phi}{R}$$

3. Lenz's Law

- The direction of the induced emf is always such that it tends to oppose the change in magnetic flux that has caused it.

4. Motional EMF

The induced emf due to the motion of electric conductor in the presence of magnetic field is called as motional emf. Motional emf can arise either by translation or rotation of a conductor.

5. Motional emf due to translation

- Consider a conductor, moving with a velocity v, in a magnetic field $\vec{B}$.
- The general expression for motional EMF across the ends of a rod having irregular shape as shown in the figure is $\Delta V = Bl'v'$
- Where l' is effective length which is equal to the shortest distance between the two end points P and Q.

  v' - Component of velocity perpendicular to l'

  B - External magnetic field which is perpendicular to both v' and l'

6. Motional emf due to rotation

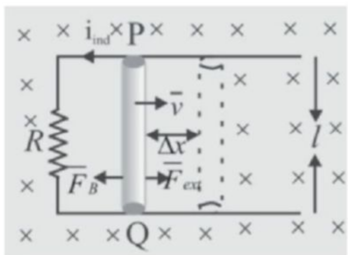
- Consider a conducting rod rotating with angular velocity ω about an axis passing through one of its ends. The length of the rod is l.
- The general expression for motional EMF across the ends of a rod which is in rotation about P.

$$\Delta V = \frac{1}{2}B\omega l'^2$$

l' is the effective length of the rod.

7. Induced EMF in a sliding Conductor

Consider a conducting rod of length l that moves on a U-shaped loop. An external force acts on the rod to move it with velocity v.



(I) Induced current

$$\text{Induced current } i_{in} = \frac{\varepsilon}{R} = \frac{Bvl}{R}$$

(II) Magnetic force on the conductor

$$F = Bi_{in}l = B\left(\frac{Bvl}{R}\right)l = \frac{B^2vl^2}{R}$$

(III) Power dissipiated in moving the conductor

$$P_{agent} = \frac{dW}{dt} = \vec{F}_{agent}.\vec{v} = \frac{B^2v^2l^2}{R}$$

(IV) Electrical power

$$P_{thermal} = i^2R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2v^2l^2}{R}$$

8. Induced Electric field

- A time varying magnetic field always produces an electric field. The electric field that will be produced is given by

$$\oint \vec{E}\times d\vec{l} = -\frac{d\phi_B}{dt}$$

9. Magnitude of $\vec{E}$

The magnetic field at all points with in the cylindrical region whose cross-section is indicated in the accompanying figure starts increasing at a constant rate 'α'. The magnitude of electric field as a function of r is given as follows.

(I) For $r < R$

$$\Rightarrow E = -\frac{r}{2}\frac{dB}{dt} = -\frac{r}{2}\alpha \Rightarrow E \propto r$$

(II) For $r = R$

$$E = -\frac{R}{2}\alpha$$

(III) For $r > R$

$$E = -\frac{R^2}{2r}\alpha \Rightarrow E_{out} \propto \frac{1}{r}$$

10. Self induction

Self inductance is defined as the induction of a voltage in a current carrying wire when the current in the wire itself is changing.

(I) Solenoid

- The mean radius of the solenoid is r.
- The magnetic field inside solenoid $B = \mu_0 nI$
- The magnetic flux linked is ϕ=NBA, where N is total number of turns in length l of solenoid.
- Self-inductance $L = \frac{\phi}{I} = \mu_0 n^2 Al = \frac{\mu_0 N^2 A}{l}$

  Where ($N=nl$)

(II) Toroid

- A toroid of very large radius r is taken so that the difference between the outer and inner radii can be neglected. The total number of turns are equal to N. The magnetic field at a distance r from the centre is

$$B = \mu_o Ni / 2\pi r$$

- Flux through the toroid

$$\phi = B(NA) \quad \phi = \frac{\mu_o N^2 iA}{2\pi r},$$

- The self-inductance

$$L = \frac{\phi}{I} = \frac{\mu_o N^2 A}{2\pi r}$$

11. Energy stored in an inductor

$$U = \int_0^i Li\, di \Rightarrow U = \frac{1}{2}Li^2$$

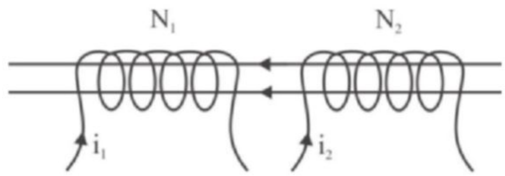
For a solenoid $L = \frac{\mu_0 N^2 A}{l}$

$$U = \frac{1}{2\mu_0}B^2 V$$

$$\frac{U}{V} = u = \frac{B^2}{2\mu_0}$$

12. Mutual Induction

- The phenomenon of production of e.m.f. in a coil when the current in neighbouring coil changes is called mutual induction.



$$\phi_1 \propto i_2 \quad \& \quad \phi_2 \propto i_1$$

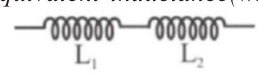
$$\phi_1 = Mi_2 \quad \& \quad \phi_2 = Mi_1$$

M - is same for a given pair of coils.

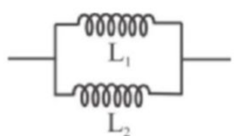
$$\varepsilon_1 = -M\frac{di_2}{dt} \;\&\; \varepsilon_2 = -M\frac{di_1}{dt}$$

13. Equivalent Inductance

(I) Equivalent inductance(when M=0)



$$L = L_1 + L_2$$



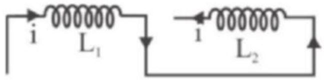
$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

(II) Equivalent inductance(when $M \neq 0$)

When fluxes support each other

$L = L_1 + L_2 + 2M$

When fluxes oppose each other



$L = L_1 + L_2 - 2M$

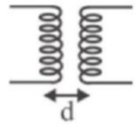
where M- Mutual inductance

$M = K\sqrt{L_1 L_2}$, K- Coupling constant

14. Coupling constant(K)

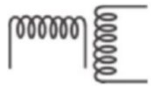
$K = 1$

Ideal coupling(Coaxial fashion)



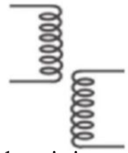
$0 < K < 1, \quad M = K\sqrt{L_1 L_2}$

If d- decreases $\Rightarrow K \to 1$



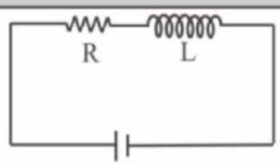
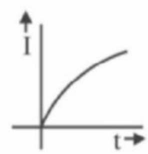
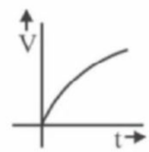
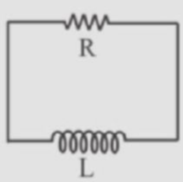
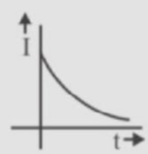
No coupling,

$K = 0 \Rightarrow M = 0$



No coupling, $K = 0 \Rightarrow M = 0$

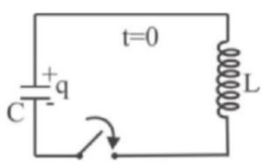
Unless it is specified always assume that M=0

L R circuits				
	current through the conductor	Time constant (τ)	I vs t	V vs t V - Potential difference
 LR - Charging circuit	$I = I_{max}\left[1 - e^{-\frac{tR'}{L}}\right]$	$\frac{L}{R'}$		
 LR - Discharging circuit	$I = I_{max} e^{-\frac{tR'}{L}}$	$\frac{L}{R'}$		

• R' is the equivalent resistance across the inductor, replace the battery with a conducting wire

15. LC Oscillations

A charged capacitor C is connected to an inductor L, the charge and current in the circuit start oscillating simple harmonically.



Energy in the capacitor is $U_C = \frac{q^2}{2C} = \frac{q_o^2 \cos^2 \omega t}{2C}$

Energy in the inductor is

$$U_L = \frac{1}{2}Li^2 = \frac{Lq_o^2\omega^2 \sin^2 \omega t}{2}$$

Total energy $T.E = U_C + U_L = \frac{q_o^2}{2C}$

Here $\omega = \frac{1}{\sqrt{LC}}$

Electromagnetic waves

16. Electric Flux

$$\phi_E = \int \bar{E}.d\bar{s}$$

- Units N - m² /Coulomb or Volt meter.

17. Displacement Current

- The rate of change of electrical flux produces a current called displacement current " i_d".
- Unlike conduction current displacement current exists when there is rate of change of electrical flux.
- The displacement current is found between the plates of a condenser during its charging or discharging.

18. Ampere -Maxwell's Law

- $\int \vec{B}.d\vec{l} = \mu_0 (i_c + i_d)$

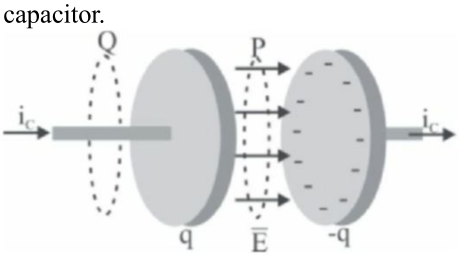
$$\Rightarrow \int \vec{B}.d\vec{l} = \mu_0 \left(i_c + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

i_c = The conduction current found in a conductor carrying current.
i_d = Displacement current which is found between the plates of a condenser which is discontinuous.

- Displacement Current : $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

19. Measurement of magnetic field

A variable current is applied to a parallel plate capacitor.



(I) *At point Q*

$$\int \vec{B}.d\vec{l} = \mu_0 (i_c + i_d)$$

- As $i_d = 0 \Rightarrow \int \vec{B}.d\vec{l} = \mu_0 i_c$

(II) *At point P:*

$$\int \vec{B}.d\vec{l} = \mu_0 (i_c + i_d)$$

But $i_c = 0 \Rightarrow \int \vec{B}.d\vec{l} = \mu_0 i_d$

20. Magnetic field between the plates

- The figure shows the cross-section of Electrical field between the plates of a condenser having the plates of radius R.
- The electric field is acting into the paper.

- The magnetic field at a distance " r " from the axis is

$$\int \vec{B}.d\vec{l} = \mu_0 i \Rightarrow B\, 2\pi r = \mu_0 \frac{i_d}{\pi R^2}.\pi r^2$$

$$\therefore B = \frac{\mu_0}{2\pi} i_d \frac{r}{R^2}$$

21. Maxwell's Equations

(1) *Gauss law for electricity*

$$\int \vec{E}.d\vec{A} = q_{net} / \varepsilon_0$$

(2) *Gauss law for magnetism*

$$\int \vec{B}.d\vec{A} = 0$$

(3) *Faraday's law*

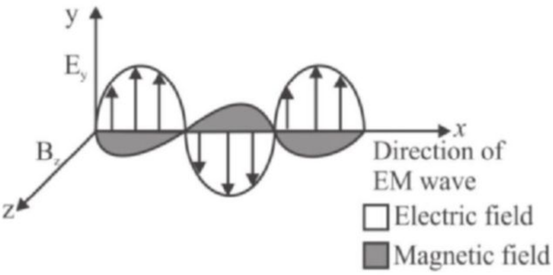
$$\int \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$$

(4) *Lorentz Force*

$$\vec{F} = q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

22. Nature of electromagnetic waves

Electric and magnetic fields oscillate sinusoidally in space and time in an electromagnetic wave,



- $\vec{E} = E_y\hat{j} = E_0 \sin[kx\text{-}\omega t]$ $(E_x = E_z = 0)$
- $\vec{B} = B_z\hat{k} = B_0 \sin[kx\text{-}\omega t]\hat{k}$ $(B_x = B_y = 0)$
- The magnitudes of $\vec{E}$ and $\vec{B}$ are related by

$$\frac{E}{B} = c \quad or \quad \frac{E_0}{B_0} = c$$

- Electromagnetic waves travel through vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \text{ x } 10^8 \text{ m/s}$$

where μ_0 = permeability of free space

ε_0 = permittivity of free space

- The Poynting vector $\vec{S} = \vec{E} \times \vec{H} = \dfrac{\vec{E} \times \vec{B}}{\mu_0}$ represents the direction of energy flow per unit area per sec along the direction of wave propagation.

23. Energy density

$$u_{avg} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2$$

24. Intensity of Radiation

$$\Rightarrow I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

25. Momentum and Radiation Pressure

The magnitude of the momentum delivered to a surface is

$$p = \frac{U}{c} \text{ (Complete absorption)}$$

where c = velocity of light
U = total energy

$$p = \frac{2U}{c} \text{ (Complete reflection)}$$

- When radiation is incident on a surface, Radiation pressure $P_r = \dfrac{I}{c}$ (total absorption) and $P_r = \dfrac{2I}{c}$ (total reflection back along the incident path)

Exercise

Magnetic Effects of Current

1. A uniform magnetic field, $B = B_0\hat{j}$ exists in space. A particle of mass m and charge q, is projected towards x-axis with speed v, from a point $(a, 0, 0)$. The maximum value of v for which the particle does not hit the yz- plane is

(a) $\dfrac{Bqa}{m}$ (b) $\dfrac{Bq}{2am}$ (c) $\dfrac{Bqa}{2m}$ (d) $\dfrac{Bq}{am}$

2. An electron of mass m and charge q is travelling with a speed v along a circular path a radius r at right angles to a uniform of magnetic field B. If speed of the electron is doubled and the magnetic field is halved, then resulting path would have a radius of

(a) $\dfrac{r}{2}$ (b) 2r (c) $\dfrac{r}{4}$ (d) 4r

3. Two concentric circular loops of radii r_1 and r_2 carry clockwise and anticlockwise currents i_1 and i_2. If the centre is a null point, i_1/i_2 must be equal to

(a) r_2/r_1 (b) r_2^2/r_1^2 (c) r_1^2/r_2^2 (d) r_1/r_2

4. A hollow cylinder having infinite length and carrying uniform current per unit length λ along the circumference. Magnetic field inside the cylinder is

(a) $\mu_0\lambda$ (b) $2\mu_0\lambda$

(c) $\dfrac{\mu_0\lambda}{2}$ (d) zero

5. A wire along x-axis carries a current 3.5A. The force on a 1 cm section of the wire exerted by the magnetic field $\overline{B} = 0.74T\,\hat{j} - 0.36T\,\hat{k}$

(a) $\left(2.59\hat{k} + 1.26\hat{j}\right)10^{-2}\,N$

(b) $\left(-1.26\hat{k} + 2.59\hat{j}\right)10^{-2}\,N$

(c) $\left(1.26\hat{k} - 2.59\hat{j}\right)10^{-2}\,N$

(d) $\left(-2.59\hat{k} - 1.26\hat{j}\right)10^{-2}\,N$

Magnetism & Matter

6. A bar magnet of moment M is bent into arc, its moment now
(a) Decreases (b) Increases
(c) Does not change (d) May change

7. A very long magnet is held vertically with its south pole on a table. A single neutral point is located on the table to the
(a) East of the magnet (b) North of the magnet
(c) West of the magnet (d) South of the magnet

8. The coil in a MCG has an area of $4\,cm^2$ and 500 turns. The intensity of magnetic induction is 2 T. When a current of $10^{-4}\,A$ is passed through it, the deflection is $20°$. The couple per unit twist is (N-m/degree)
(a) 2×10^{-6} (b) 3×10^{-6} (c) 5×10^{-6} (d) 4×10^{-6}

9. Due to the earth's magnetic field, charged cosmic ray particles
(a) Can never reach the equator
(b) Can never reach the poles
(c) Require less kinetic energy to reach the equator than the poles
(d) Require greater kinetic energy to reach the equator than the poles

10. A magnet is placed on a paper in a horizontal plane for locating neutral points. A dip needle placed at the neutral point will be horizontal at the
(a) Magnetic equator (b) Magnetic pole
(c) Latitude angle of 60° (d) Latitude angle 45°

Electro Magnetic Induction

11. A coil of 40Ω resistance has 100 turns and radius 6 mm is connected to an ammeter of resistance of 160Ω. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μC charge flows through it. The intensity of magnetic field will be
(a) 5.66 T (b) 6.55 T
(c) 0.566 T (d) 0.655 T

12. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$; $B = B_0 \sin(kx - \omega t)$. Which of the following is true?
(a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$
(c) $E_0 k = 2B_0 \omega$ (d) $E_0 B_0 = \omega k$

13. An electric field of 300V/m is confined to a circular area 10 cm in diameter. If the field is increasing at the rate of 20V/m-s, the magnitude of magnetic field at a point 15cm from the center of the circle will be-
(a) 1.85×10^{-17} T (b) 1.85×10^{-15} T
(c) 1.85×10^{-16} T (d) 1.85×10^{-18} T

14. A magnet is dropped down inside a long vertical copper tube
(a) The magnet moves with continuously increasing velocity and ultimately acquires a constant terminal velocity
(b) The magnet moves with continuously decreasing velocity and ultimately comes to rest
(c) The magnet moves with continuously increasing velocity but constant acceleration
(d) The magnet moves with continuously increasing velocity and acceleration

15. Which of the following is constructed on the principle of electromagnetic induction?
(a) Voltmeter (b) Galvanometer
(c) Generator (d) Electric motor

ANSWER KEY

Magnetic Effects of Current

1. a 2. d 3. d 4. d 5. a

Magnetism & Matter

6. a 7. b 8. a 9. a 10. a

Electro Magnetic Induction

11. c 12. a 13. d 14. a 15. c

Previous years NEET/AIPMT Questions

ELASTICITY

1. Two wires are made of the same material and have the same volume. The first wire has cross- sectional area A and the second wire has cross-sectional area 3A. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount ? **[2018]**
(a) 4 F (b) F (c) 6 F (d) 9 F

2. The Young's modulus of steel is twice that of brass. Two wires of the same length and of the same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of **[2015]**
(a) 1 : 1 (b) 1 : 2 (c) 4 : 1 (d) 2 : 1

3. The approximate depth of an ocean is 2700 m. The compressibility of water is $45.4 \times 10^{-11}\,Pa^{-1}$ and density of water is $10^3\,kg/m^3$. What frictional compression of water will be obtained at the bottom of the ocean? **[2015]**
(a) 1.2×10^{-2} (b) 1.4×10^{-2}
(c) 0.8×10^{-2} (d) 1.0×10^{-2}

4. Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. Which of the following graphs is a straight line? **[2014]**
(a) Δl *versus* $\frac{1}{l}$ (b) Δl *versus* l
(c) Δl *versus* l^2 (d) Δl *versus* $\frac{1}{l^2}$

5. The following four wires are made of same material. Which of these will have the largest extension when the same tension is applied? **[2013]**
(a) Length = 300 cm, diameter = 3 mm
(b) Length = 50 cm, diameter = 0.5 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 100 cm, diameter = 1 mm

FLUID MECHANICS

1. Two non-mixing liquid of densities ρ and $n\rho\ (n > 1)$ are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length $pL (p < 1)$ in the denser liquid. The density d is equal to. **[2016]**
(a) $\{1 + (n+1)p\}\rho$ (b) $\{2 + (n+1)p\}\rho$
(c) $\{2 + (n-1)p\}\rho$ (d) $\{1 + (n-1)p\}\rho$

2. The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is v, the speed of the ejection of the liquid through the holes is **[2015]**

(a) $\frac{v^2 R}{nr}$ (b) $\frac{vR^2}{n^2 r^2}$ (c) $\frac{vR^2}{nr^2}$ (d) $\frac{vR^2}{n^3 r^2}$

3. A wind with speed 40 m/s blows parallel to the roof of a house. The area of the root is $250 m^2$.
Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be $\left(P_{air} = 1.2 kg/m^2\right)$ **[2015]**

(a) 4.8×10^5 N, downwards
(b) 4.8×10^5 N, upwards
(c) 2.4×10^5 N, upwards
(d) 2.4×10^5 N, downwards

SURFACE TENSION

1. Water rises to height 'h' in capillary tube. If the length of capillary tube above the surface of water is made less than 'h' then **[2015]**
(a) Water rises up to the top of capillary tube and stays there without overflowing.
(b) Water does not rise at all.
(c) Water rises up to a point a little below the top and stays there.
(d) Water rises up to the top of capillary tube and then starts overflowing like a fountain.

2. A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then **[2014]**

(a) Energy $= 4VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released

(b) Energy $= 3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released

(c) Energy $= 3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed

(d) Energy is neither released nor absorbed

3. The wettability of a surface by a liquid depends primarily on **[2013]**
(a) Angle of contact between the surface and the liquid
(b) Density
(c) Viscosity
(d) Surface tension

4. Radius of a soap bubble is increased from R to 2R. Work done in this process in terms of surface tension is **[2004]**
(a) $24\pi R^2 S$ (b) $48\pi R^2 S$
(c) $12\pi R^2 S$ (d) $36\pi R^2 S$

5. A capillary tube of radius R is immersed in water and water rises in it to a height H. Mass of water in the capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will now be **[2002]**
(a) 2M (b) M (c) M/2 (d) 4M

ANSWER KEY

ELASTICITY

1. d 2. d 3. a 4. c 5. b

FLUID MECHANICS

1. d 2. c 3. c

SURFACE TENSION

1. a 2. b 3. a 4. a 5. a

HINTS & SOLUTIONS

ELASTICITY

1.Sol: Volume V

$= A \times L_1 = 3A \times L_2$

$L_2 = \frac{L_1}{3}$

Stress $= Y(strain)$

$F = \left(\frac{AY}{L}\right)\Delta L;$

$F' = \left[\frac{3AY}{\left(\frac{L}{3}\right)}\right] \cdot \Delta L = 9F$

2.Sol: Young's modulus $Y=\frac{W\cdot l}{A\Delta l}$

l, A and Δl are the same for both the wires.

$Y \propto W$

$$\frac{Y_s}{Y_b}=\frac{W_s}{W_b}=\frac{2}{1}$$

$$W_s : W_b = 2:1$$

3.Sol: $$B=\frac{1}{\text{Compressibility}}$$

$$\frac{1}{\text{compressibility}}=\frac{h\rho g}{\left(\frac{\Delta V}{V}\right)}$$

$$\left(\frac{\Delta V}{V}\right)=(h\rho g)\text{Compressibility}$$

$$\Rightarrow \frac{\Delta V}{V}=2.7\times10^3\times10^3\times10\times45.4\times10^{-11}$$

$$=1.23\times10^{-2}$$

4.Sol: Young's modulus is given by

$$Y=\frac{F\times l}{A\times\Delta l} \quad (\because \text{ Hooke's Law}) \quad \text{(i)}$$

As $V = A\times l = constant$ (ii)

From Eqs. (i) and (ii), we get

$$Y=\frac{F\times l^2}{V\times\Delta l}$$

$$\Rightarrow \Delta l=\frac{F}{V\times Y}\times l^2$$

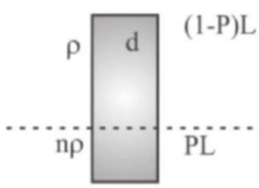
$$\Rightarrow \Delta l \propto l^2$$

5.Sol: As $Y=\frac{F\times L}{\Delta L\times A}=\frac{mg.L}{\Delta L\cdot A}$ or $\Delta L=\frac{mgL}{YA}$

$\Rightarrow \Delta L \propto \frac{L}{A}$, which is maximum for option (b)

FLUID MECHANICS

1.Sol: The cylinder present in two liquids are shown in the figure.



As the cylinder is in equilibrium $F_g=F_{b_1}+F_{b_2}$

$$LAdg=(pL)A(n\rho)g+(1-p)LA\rho g$$

$$\Rightarrow d=(1-p)\rho+pn\rho=[1+(n-1)p]\rho$$

2.Sol: From equation of continuity

$$\pi R^2 v=n\pi r^2 v_1 \Rightarrow v_1=\frac{vR^2}{nr^2}$$

3.Sol: Applying Bernoulli's theorem just above and just below the roof,

$$P+\frac{1}{2}\rho v^2=P_0+0 \qquad (P_0-P)=\Delta P=\frac{1}{2}\rho v^2$$

Hence lift of the roof

$$F=\Delta P.A=\frac{1}{2}\rho A v^2$$

SURFACE TENSION

2.Sol: Let R is the radius of the bigger drop and N drops are combined to form the bigger drop.

$$N\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$$

$$Nr^3=R^3 \quad \text{(i)}$$

The energy released

$$E=T\left[A_i-A_f\right]$$

$$E=T\left[N4\pi r^2-4\pi R^2\right]$$

$$=4\pi R^2 T\left[\frac{Nr^2}{R^2}-1\right]$$

$$=3\left(\frac{4}{3}\pi R^3\right)T\left[\frac{Nr^2}{R^3}-\frac{1}{R}\right]$$

$$=3VT\left[\frac{1}{r}-\frac{1}{R}\right]$$

4.Sol:

$$W=8\pi S(R_2^2-R_1^2)=8\pi S[(2R)^2-R^2]=24\pi R^2 S$$

5.Sol: Mass of liquid in capillary tube

$$M=\pi R^2 h\times\rho=\pi R^2\rho\frac{2T\cos\theta}{\rho Rg}$$

$\therefore$ $M\propto R$. If radius becomes double, then mass will become twice.

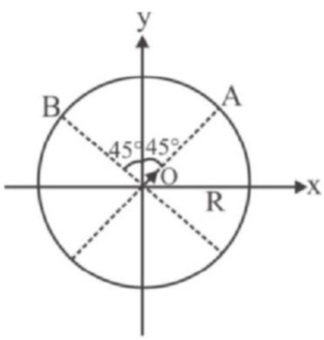
SOLVED PAPER ★ Physics ★

JEE ADVANCED 2019

PAPER-2

SECTION-1
More than one answer type questions

1. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are: (ϵ_0 is permittivity of free space R>>dipole size)



(a) The magnitude of total electric field on any two points of the circle will be same.

(b) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(c) $R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

(d) Total electric field at point B is $\vec{E}_B = 0$

2. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical? [g is the acceleration due to gravity]

(a) The angular acceleration of the rod will be $\frac{2g}{L}$

(b) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

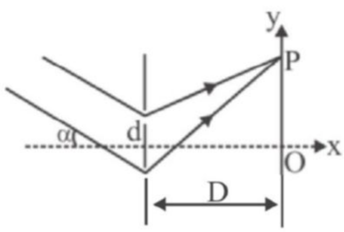
(c) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(d) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

3. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n = 1 to the state n = 4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the

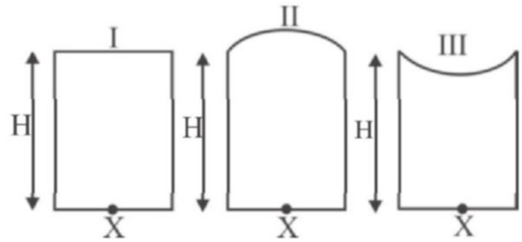
change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e respectively. If $\lambda_a / \lambda_e = 1/5$, which of the option(s) is/are correct?
[Use hc = 1242 eV nm ; 1nm = 10^{-9} m, h and c are Plank's constant and speed of light, respectively]
(a) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is 1/4
(b) m = 2
(c) $\Delta p_a / \Delta p_e = 1/2$
(d) $\lambda_e = 418$

4. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m.
A parallel beam of light of wavelength 600 nm is incident on the slits at angle ? as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/ are correct?



(a) For $\alpha = 0$, there will be constructive interference at point P.
(b) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.
(c) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.
(d) Fringe spacing depends on α.

5. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinders, respectively, the correct statement(s) is/are:

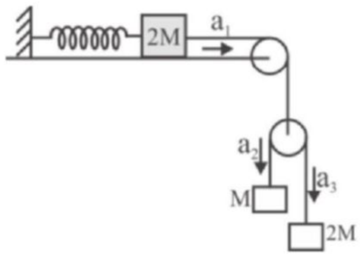


(a) $H_2 > H_1$
(b) $H_3 > H_1$
(c) $0.8\,cm < (H_2 - H_1) < 0.9\,cm$
(d) $H_2 > H_3$

6. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume V0/4, then the correct statement(s) is/are,
(Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)
(a) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$.
(b) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$
(c) Adiabatic constant of the gas mixture is 1.6
(d) The work |W| done during the process is $13RT_0$

7. A block of mass 2M is attached to a massless spring with spring–constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of

the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



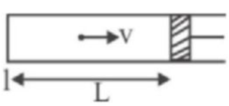
(a) At an extension of $\frac{x_0}{4}$ of the spring. The magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(b) $x_0 = \frac{4Mg}{k}$

(c) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(d) $a_2 - a_1 = a_1 - a_3$

8. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}v_0$.
Where dL is the infinitesimal displacement of the piston.
Which of the following statement(s) is/are correct?

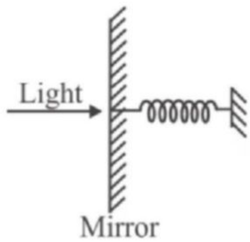


(a) After each collision with the piston, the particle speed increases by 2V.
(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$
(c) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$
(d) The rate at which the particle strikes the piston is v/L

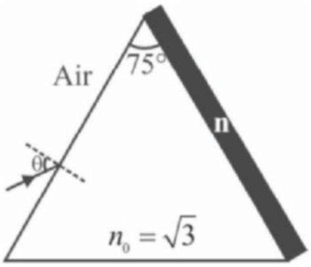
SECTION-2
Integer type questions

1. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is ____

2. A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring–mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24}\,m^{-2}$ with h as Planck's constant. N photons of wavelength $\lambda = 8\pi \times 10^{-6}\,m$ strike the mirror simultaneously at normal incidence such that the mirror gets displaced by $1\mu m$. If the value of N is $x \times 10^{12}$, then the value of x is _____. [Consider the spring as massless]

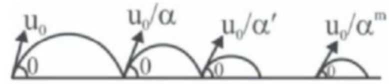


3. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is ______.

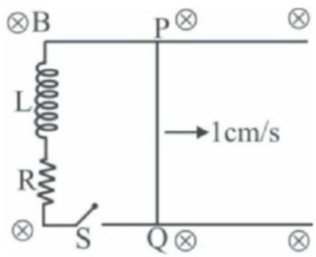


4. Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α – decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ –decay. The energy of the emitted γ photon is _____keV.
[Given : atomic mass of $^{226}_{88}Ra = 226.005u$, atomic mass of $^{222}_{88}Rn = 222.0050u$, atomic mass of α particle = 4.000 u, 1u=931MeV/c^2, c is speed of light]

5. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α . Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is ___________



6. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1mH and a resistance $R = 1\Omega$ as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B=1T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 milli second is x × 10^{-3}A, where the value of x is _____________.
[Assume the velocity of wire PQ remains constant (1cm/s) after key S is closed. Given : e^{-1} = 0.37, where e is base of the natural logarithm]



SECTION-3

Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings 1, 2, 3 and 4 with mass per unit length $1\mu, 2\mu, 3\mu$ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1(μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I	List-II
(I) String-1 (μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $\frac{1}{\sqrt{2}}$

(IV) String-4 (4μ) (S) $\frac{1}{\sqrt{3}}$

(T) 3/16

(U) 1/16

1. If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be :
(a) $I \rightarrow P, II \rightarrow Q, III \rightarrow T, IV \rightarrow S$
(b) $I \rightarrow P, II \rightarrow R, III \rightarrow S, IV \rightarrow Q$
(c) $I \rightarrow Q, II \rightarrow S, III \rightarrow R, IV \rightarrow P$
(d) $I \rightarrow Q, II \rightarrow P, III \rightarrow R, IV \rightarrow T$

2. The length of the strings 1, 2, 3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ respectively. Strings 1, 2, 3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be :
(a) $I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow U$
(b) $I \rightarrow P, II \rightarrow Q, III \rightarrow R, IV \rightarrow T$
(c) $I \rightarrow P, II \rightarrow Q, III \rightarrow T, IV \rightarrow U$
(d) $I \rightarrow T, II \rightarrow Q, III \rightarrow R, IV \rightarrow U$

Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas

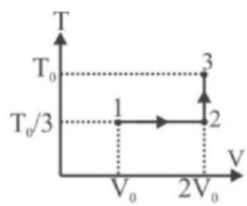
$$X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right).$$

Here, R is gas constant, V is volume of gas. T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

LIST-I	LIST-II
(I) Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3 + \ln 2)$
	(U) $\frac{5}{6}RT_0$

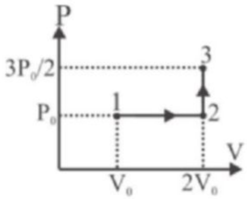
3. If the process on one mole of monatomic ideal gas is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$ the correct match is,



(a) $I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow S$
(b) $I \rightarrow P, II \rightarrow T, III \rightarrow Q, IV \rightarrow T$
(c) $I \rightarrow S, II \rightarrow T, III \rightarrow Q, IV \rightarrow U$
(d) $I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow P$

4. If the process carried out on one mole of monoatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$ the correct

match is



(a) $I \to S, II \to R, III \to Q, IV \to T$

(b) $I \to Q, II \to R, III \to P, IV \to U$

(c) $I \to Q, II \to S, III \to R, IV \to U$

(d) $I \to Q, II \to R, III \to S, IV \to U$

ANSWER KEY

SECTION-1

1. (c,d) **2.** (b,c&d) **3.** (a,b) **4.** (b)
5. (a,d) **6.** (a,c,d) **7.** d **8.** (a,c)

SECTION-2

1. 0.69 **2.** 1 **3.** 1.5 **4.** 135
5. 4 **6.** 0.63

SECTION-3

1. b **2.** c **3.** d **4.** d

HINTS & SOLUTIONS

SECTION-1

1.Sol: R>> dipole size

So the circle is equipotential

So, E_{net} Should be $\perp$ to surface so

$$\frac{p_0}{4\pi\varepsilon_0 r^3} = E_0 \Rightarrow r = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$$

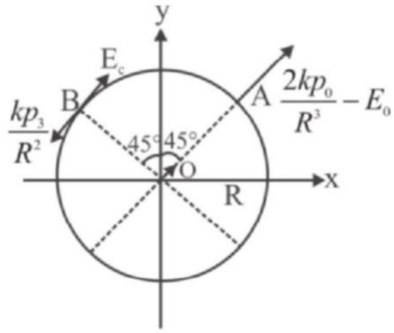
At point B net electric field will be zero.

$$E_B = 0$$

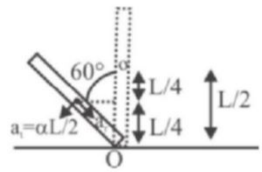
$$(E_A)_{Net} = \frac{2kp_0}{r^3} + E_0 = 3E_0$$

Electric field at point A $\vec{E}_A = \frac{3}{\sqrt{2}} E_0[\hat{i} + \hat{j}]$

$$(E_B)_{Net} = 0$$



2.Sol:



$$\Delta K + \Delta U = 0$$

$$\frac{1}{2} I_0 \omega^2 = -\Delta U$$

$$\frac{1}{2}\frac{ML^2}{3}\omega^2 = -\left(-Mg\frac{L}{4}\right)$$

$$\omega = \sqrt{\frac{3g}{2L}}$$

$$\Rightarrow a_{radial} = \omega^2 \frac{L}{2} = \frac{3g}{2L}\frac{L}{2} = \frac{3g}{4} \quad \Rightarrow \quad \tau = I_0 \alpha$$

$$\alpha = \frac{Mg\frac{L}{2}\sin 60^\circ}{M\frac{L^2}{3}} = \frac{3\sqrt{3}g}{4L}$$

The acceleration of CM of the rod along vertical direction is

$$\Rightarrow a_v = \left(\alpha\frac{L}{2}\right)\sin 60^\circ + \omega^2 \frac{L}{2}\cos 60^\circ$$

$$\Rightarrow a_v = \frac{3\sqrt{3}g}{8}\frac{\sqrt{3}}{2} + \frac{3g}{8}$$

$$\Rightarrow a_v = \frac{15g}{16}$$

$$\Rightarrow \quad Mg - N = Ma_v \Rightarrow \quad N = \frac{Mg}{16}$$

3.Sol: As $E \alpha \frac{1}{\lambda}$ & $E = \frac{-13.6eV}{n^2}$

$$\lambda_a / \lambda_e = \frac{E_4 - E_m}{E_4 - E_1} = \frac{\frac{1}{m^2} - \frac{1}{16}}{1 - \frac{1}{16}} = \frac{1}{5}$$

On solving we get

$m = 2$

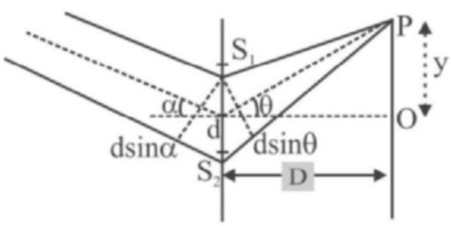
$$\lambda_e = \frac{1242 \times 16}{13.6 \times 3} \approx 487\,nm$$

As $\quad K \alpha \frac{1}{n^2}$

$\frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$ as kinetic energy is proportional

$\frac{1}{n^2}$

4.Sol:



The net path difference Δx is

$$\Delta x = d \sin \alpha + d \sin \theta$$

$$\sin \theta \simeq \tan \theta = \frac{y}{b}$$

$$\Delta x = d\alpha + \frac{dy}{D}$$

If $\alpha = 0$

$$\therefore \quad \Delta x = dy/D = \frac{0.3 \times 11}{1000} = 33 \times 10^{-4}\,mm$$

Δx interms of $\lambda = \frac{33 \times 10^{-4}}{600 \times 10^{-6}} \lambda = \frac{11\lambda}{2}$

So destructive interference takes place

At Point P

$$\Delta x = 0.3\,\text{mm} \times \frac{0.36}{\pi} \times \frac{\pi}{180} + \frac{0.3\,\text{mm} \times 11\,\text{mm}}{1000}$$

$$= 39 \times 10^{-4}\,\text{mm}$$

$$39 \times 10^{-4} = (2n - 1) \times \frac{600 \times 10^{-9} \times 10^3}{2}$$

$n = 7$

There will be destructive interference

At Point O

$$\Delta x = 3\,\text{mm} \times \frac{0.36}{\pi} \times \frac{\pi}{180} + 0 = 600\,\text{nm}$$

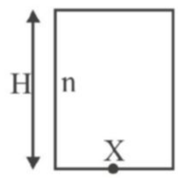
$600\,\text{nm} = n\lambda$

$n = 1$

Constructive interference

Fringe width does not depend on α.

5.Sol: Case-I:

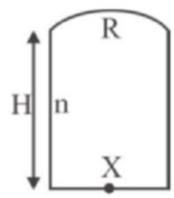


$H = 30\,cm$

$n = 3/2$

$$H_1 = H/n \Rightarrow \frac{30 \times 2}{3} = 20\,\text{cm}$$

Case-II:



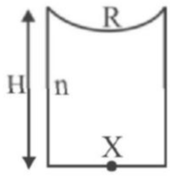
$R = 300\,cm$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{-H_2} - \frac{3}{-2 \times 30} = \frac{1 - \frac{3}{2}}{-300}$$

$$H_2 = \frac{600}{29} = 20.684\,cm$$

Case-III:



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R};$$

$$\frac{1}{-H_3} - \frac{3}{-2\times 30} = \frac{1-\frac{3}{2}}{300};$$

$$H_3 = \frac{600}{31} = 19.354\,cm$$

6.Sol: $\gamma_{mix} = \dfrac{n_1C_{P1} + n_2C_{P2}}{n_1C_{V1} + n_2C_{V2}} = \dfrac{8}{5}$

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$P_0V_0^{8/5} = P_2\left(\frac{V_0}{4}\right)^{8/5}$$

$$P_2 = 9.2P_0$$

$$W = \frac{P_0V_0 - 9.2P_0\frac{V_0}{4}}{3/5} = -13RT_0$$

$$\therefore \quad |W| = 13RT_0$$

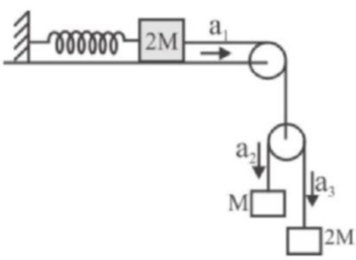
$$T_1V_1^{\gamma-1} = T_2V_{\gamma-1}$$

$$T_2 = T_1(2)^{6/5} = 2.3T_0$$

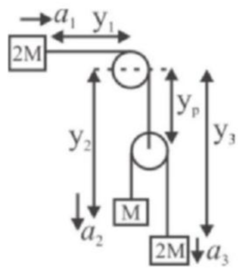
Average kinetic energy of gas mixture

$$= nC_{V_{mix}}T_2 = 23RT_0$$

7.Sol:



Let y_1, y_2 & y_3 are the positions of the three blocks w.r.t the fixed pulley.



The length of the first string is

$$l_1 = y_1 + y_p \qquad (1)$$

The length of the second string is

$$l_2 = y_2 - y_p + y_3 - y_p \qquad (2)$$

From eq's 1 & 2

$$y_2 + y_3 + 2y_1 = \text{constant}$$

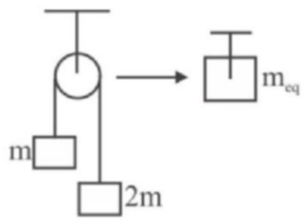
On differentiating w.r.to t

$$\frac{d^2y_2}{dt^2} + \frac{d^2y_3}{dt^2} + \frac{2d^2y_1}{dt^2} = 0$$

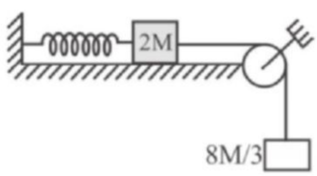
$$a_2 + a_3 + 2(-a_1) = 0$$

$$\Rightarrow a_1 - a_3 = a_2 - a_1$$

For other options use m equivalent



$$m_{eq} = \frac{4(m_1)(m_2)}{m_1 + m_2} = \frac{4(M)(2M)}{M + 2M} = \frac{8M}{3}$$



The maximum extension in the spring is

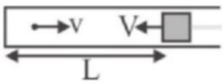
$$\frac{1}{2}kx_0^2 = \frac{8Mg}{3}x_0$$

$$x_0 = \frac{16Mg}{3k}$$

$$v = \omega\sqrt{x_0^2 - x^2} = \omega\sqrt{x_0^2 - \frac{x_0^2}{4}}$$

$$v = \frac{\sqrt{3}}{2}\omega x_0 = \frac{\sqrt{3}}{2}\times\frac{16Mg}{3K}\sqrt{\frac{3K}{14M}} = g\sqrt{\frac{32M}{K}}$$

$$a_{\frac{x_0}{4}} = \frac{x_0}{4}\omega^2 = \frac{x_0}{4}\frac{3K}{14M} = \frac{2g}{7}$$

8.Sol: 

The rate of collision of the particle with the piston is $\frac{1}{2L/\text{v}} = \frac{\text{v}}{2L}$

The speed of the particle after collision with the piston is $\text{v} + 2V$

If the piston moves inward by dL the speed of the particle increases by

$$d\text{v} = 2V \times \frac{dL}{V} \times \frac{\text{v}}{2L}$$

$$\Rightarrow \quad \frac{d\text{v}}{\text{v}} = \frac{dL}{L}$$

As $K = \frac{1}{2}m\text{v}^2 \Rightarrow \frac{dK}{K} = \frac{2d\text{v}}{\text{v}} = \frac{-2dL}{L}$

$$\int\frac{dK}{K} = -2\int\frac{dL}{L}$$

$$\ln K = -2\ln L + \ln C$$

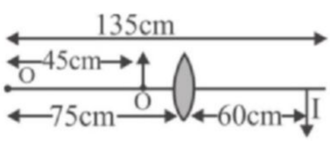
$$\ln(KL^2) = \ln C$$

$$\Rightarrow \quad KL^2 = \text{constant}$$

$$\Rightarrow \quad K_f = 4K_i$$

Correct options are (a,c)

SECTION-2

1.Sol: 

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2}\Delta v + \frac{1}{u^2}\Delta u = -\frac{1}{f^2}\Delta f$$

$$\left(\frac{\Delta f}{f^2}\right)_{max} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2}$$

$$\frac{\Delta f}{(20)^2} = \frac{1}{4(60\times 60)} + \frac{1}{4(30)(30)}$$

where $\Delta v = \Delta u = \frac{1}{4}cm$

$$\Delta f = \frac{5}{36}$$

$$\frac{\Delta f}{f} = \frac{5}{36}\times\frac{1}{20}$$

$$\frac{\Delta f}{f}\times 100\% = \frac{5}{36}\times\frac{1}{20}\times 100\% = \frac{25}{36}\%$$

$$= 0.69\%$$

2.Sol: Momentum transferred to the mirror

$$= \frac{2Nh}{\lambda}$$

$$\frac{2Nh}{\lambda} = MV_{(mean\ position)}$$

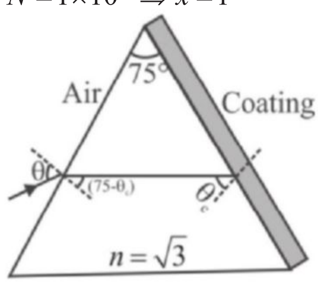
$V_{(mean\ position)} = \Omega A$ (where $A = 1\,\mu m$)

$\frac{2Nh}{\lambda} = M\Omega A$ (where $\lambda = 8\pi\times 10^{-6}$)

$$N = \frac{M\Omega(10^{-6})\lambda}{2h} = \frac{M\Omega 8\pi\times 10^{-6}\times 10^{-6}}{2h}$$

$$N = \frac{4\pi M\Omega}{h}\times 10^{-12}$$

$$N = 1\times 10^{12} \Rightarrow x = 1$$

3.Sol: 

For T.I.R at coated surface

$$\sin\theta_c = \frac{n}{\sqrt{3}}$$

Appling snell's law at first surface

$$\sin\theta = \sqrt{3}\sin(75-\theta_c)$$

$$\sin 60 = \sqrt{3}\sin(75-\theta_c)$$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75-\theta_c)$$

$$\frac{1}{2} = \sin(75-\theta_c)$$

$$\Rightarrow 30 = 75-\theta_c \Rightarrow \theta_c = 45°$$

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow n^2 = \frac{3}{2} = 1.50$$

4.Sol:

$$\Delta m = 226.005 - 222.000 - 4.000 = 0.005\,\text{amu}$$

$\therefore$ Q value $= 0.005\times 931.5 = 4.655\,MeV$

Also $\frac{K.E_\alpha}{K.E_{Rn}} = \frac{m_{Rn}}{m_\alpha}$

$$\Rightarrow K.E_{Rn} = \frac{m_\alpha}{m_{Rn}}\cdot K.E_\alpha = \frac{4}{222}\times 4.44$$

$$= 0.08\,MeV$$

$\therefore$ Energy of γ – Photon $= 4.655 - (4.44 + 0.08)$

$$= 0.135\,MeV = 135\,KeV$$

5.Sol: For the first collision

$$<V> = \frac{R}{T} = u_{x1} = V_1$$

For the total journey

$$<V>(n_{-1}) = \frac{R_1 + R_2 + ... + R_m}{T_1 + T_2 + ... + T_m}$$

$$= \frac{\frac{2u_{x_1}u_{y_1}}{g} + \frac{2u_{x_2}u_{y_2}}{g} + ... + \frac{2u_{x_m}u_{y_m}}{g}}{\frac{2u_{y_1}}{g} + \frac{2u_{y_2}}{g} + ... \frac{2u_{y_m}}{g}}$$

$$u_{x_1}\left[\frac{1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ... \frac{1}{\alpha^{2m}}}{1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ... + \frac{1}{\alpha^m}}\right] = 0.8 v_1$$

$$\Rightarrow \frac{V_1\left[\frac{1}{1-\frac{1}{\alpha^2}}\right]}{\left[\frac{1}{1-\frac{1}{\alpha}}\right]} = 0.8\,V_1 \Rightarrow \frac{\alpha}{1+\alpha} \Rightarrow \alpha = 4$$

6.Sol: $\varepsilon = Bvl = 10^{-2}\times 1\times 10^{-1}$

$$\varepsilon = 10^{-3}\,volt$$

$$i = \frac{\varepsilon}{R}\left(1 - e^{\frac{-Rt}{L}}\right) = \frac{10^{-3}}{1}(1-e^{-1})$$

$$i = 10^{-3}(1-0.37)$$

$$i = 0.63\,mA$$

SECTION-3

1.Sol: Fundamental frequency is maximum when length is minimum i.e. L_0,

Case1: $L = L_0, T = T_0, f = f_0; f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

Case 2: $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}} = \frac{f_0}{\sqrt{3}}$

Case 3: $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}} = \frac{f_0}{\sqrt{3}}$

Case 4: $f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}} = \frac{f_0}{2}$

2.Sol: Case1: $L = L_0, T = T_0, f = f_0$

$$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

Case 2: $L = \frac{3L_0}{2}$

$$f_2 = \frac{3}{2 \times \frac{3L_0}{2}}\sqrt{\frac{T_2}{2\mu}} = f_0$$

$$\Rightarrow \quad T_2 = \frac{T_0}{2}$$

Case 3: $L = \frac{5L_0}{4}$

$$f_3 = \frac{5}{2 \times \frac{5L_0}{4}}\sqrt{\frac{T_3}{3\mu}} = f_0$$

$$\Rightarrow T_3 = \frac{3T_0}{16}$$

Case 4: $L = \frac{7L_0}{4}$

$$\Rightarrow \quad f_4 = \frac{14}{2 \times \frac{7L_0}{4}}\sqrt{\frac{T_4}{4\mu}} = f_0$$

$$\Rightarrow \quad \Rightarrow T_4 = \frac{T_0}{16}$$

3.Sol: 1-2 process is isothermal and 2-3 process is isochoric.

(I) $W_{1\to2} = nRT \ln\frac{V_f}{V_i} = 1 \times R\frac{T_0}{3}\ln\frac{V_2}{V_1} = \frac{RT_0}{3}\ln 2$

$$W_{2\to3} = 0$$

$$W_{1\to2\to3} = W_{1\to2} + W_{2\to3} = \frac{RT_0}{3}\ln 2$$

(II) $\Delta U = \frac{f}{2}nR(T_f - T_i)$

$$\Delta U_{1\to2\to3} = \frac{3}{2}\left[1 \times R\left(T_0 - \frac{T_0}{3}\right)\right] = RT_0 \quad (II \to R)$$

$Q_{1\to2\to3} = \Delta U_{1\to2\to3} + W_{1\to2\to3}$ (First law of thermodynamics)

$$= RT_0 + \frac{RT_0}{3}\ln 2$$

$$= \frac{RT_0}{3}[3 + \ln 2] \quad (III \to T)$$

(IV) $Q_{1\to2} = \Delta U_{1\to2} + W_{1\to2}$

$$= 0 + \frac{RT_0}{3}\ln 2 = \frac{RT_0}{3}\ln 2 \quad (IV \to P)$$

4.Sol: (I) $W_{1\to2\to3} = W_{1\to2} + W_{2\to3}$

$$= P_0[2V_0 - V_0] + 0 = P_0V_0$$

$$W_{1\to2\to3} = P_0V_0 = \frac{RT_0}{3} \quad (I \to Q)$$

(II) $U_{1\to2\to3} = \frac{3}{2}\left[\frac{3P_0}{2} \times 2V_0 - P_0V_0\right]$

$$= \frac{3}{2} \times 2P_0V_0 = 3P_0V_0 = RT_0 \quad (II \to R)$$

$$Q_{1\to2\to3} = U_{1\to2\to3} + W_{1\to2\to3}$$

$$= RT_0 + \frac{RT_0}{3} = \frac{4RT_0}{3} \quad (III \to S)$$

(IV) $Q_{1\to2} = nC_P\Delta T$

$$= n\frac{5}{2}R(T_2 - T_1)$$

$$= \frac{5}{2}[P_0 2V_0 - P_0V_0]$$

$$= \frac{5}{6}R_0T_0 \quad (IV \to U)$$

ASSERTION & REASON

PRACTICE CORNER

Read the assertion and reason carefully to mark the correct options given below:

(a) If both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

(b) If both Assertion and Reason are true but Reason is not the correct explanation of the Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but the Reason is true.

WAVES ON STRING

1. **Assertion:** A wave of frequency 500 Hz is propagating with a velocity of $350 ms^{-1}$.
   Distance between two particles with 60° phase difference is 12 cm.
   Reason: $x = \frac{\lambda}{2\pi}\phi.$
2. **Assertion:** In a small segment of string carrying sinusoidal wave, total energy is conserved.
   Reason: The stretch in the string is maximum at the mean position
3. **Assertion:** Solids can support both longitudinal and transverse waves but only longitudinal waves can propagate in gases.
   Reason: For the propagation of transverse waves, medium must also necessarily have the property of rigidity.
4. **Assertion:** A standing wave pattern is formed in a string. The power transfer throught a point (other node and antinode) is zero always.
   Reason: At antinode displacement is maximum
5. **Assertion:** Velocity of particles, while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.
   Reason: Amplitude of vibration at antinodes is maximum and at nodes, the amplitude is zero, and all particles between two successive nodes cross the mean position together.
6. **Assertion:** In a stationary wave, there is no transfer of energy.
   Reason: There is no outward motion of the disturbance from one particle to adjoing particle in a stationary wave.
7. **Assertion:** The equation of a stationary wave is $y = 20\sin\frac{\pi x}{4}\cos\omega t.$ The distance between two consecutive anti-nodes will be 4 m.
   Reason: The data is insufficient.
8. **Assertion:** Soldiers are asked to break steps while crossing the bridge.
   Reason: The frequency of marching may be equal to the natural frequency of bridge and may lead to resonance which can break the bridge.

SOUND WAVES

1. **Assertion :** Sound, cannot propagate in the vaccum.
   Reason: Sound is a square wave.
   It propagates in a medium by a virtue of damping oscillation.
2. **Assertion:** Two persons on the surface of moon cannot talk to each other.
   Reason: There is no atmosphere on moon.
3. **Assertion:** Sound would travel faster on a hot summer day than on a cold winter day.
   Reason: Velocity of sound is directly proportional to the square of its absolute temperature.

4. Assertion: The flash of lightening is seen before the sound to thunder is heard.
Reason: Speed of sound is greater than speed of light.

5. Assertion: Sound travels faster in solids than gases.
Reason: Solid possessses greater density than gases.

ANSWER KEY

WAVES ON STRING

1. a	2. d	3. a	4. d
5. a	6. b	7. c	8. a

SOUND WAVES

1. c	2. a	3. c	4. c	5. b

HINTS & SOLUTIONS

WAVES ON STRING

1.Sol: As, $\lambda = \frac{v}{n} = \frac{350}{500} = 0.7m$

$$\phi = \frac{\pi}{3} rad \quad \text{As,} \quad x = \frac{\lambda}{2\pi}\phi$$

$$\therefore x = \frac{0.7}{2\pi} \times \left(\frac{60\pi}{180}\right) = 0.12m = 12cm$$

2.Sol: Every small segment is acted upon by forces from both sides of it hence energy is not conserved, rather it is transmitted by the element.

3.Sol: For the propagation of transverse waves, medium must have the property of rigidity. Because gases have no rigidity, (they do not possess shear elasticity), and hence transverse waves cannot be produced is gases. On the other hand, the solids possess both vlolume and shear elasticity and likewise bith the longitudinal and transverse waves can be transmitted throught them.

4.Sol: At node $v = 0$, at antinode Tension $\perp$ to velocity

$\therefore$ at both the points power $= 0 \quad (P = \vec{F} \cdot \vec{v})$

At other points $P \neq 0$.

6.Sol: In stationary wave, total energy associated with it is twice the energy of each of incidence and reflected wave. Large amount of energy is trapped with the waves. Hence, there is no transmission of energy through the waves.

7.Sol: The equation of stationary waves is

$$y = 20 \sin \frac{\pi x}{4} \cos \omega t$$

Compare with $y = 2a \sin kx \cos \omega t$

$$k = \frac{\pi}{4} \Rightarrow \lambda = \frac{2\pi}{\pi/4} = 8m$$

Distance between two consecutive antinodes

$$= \frac{\lambda}{2} = \frac{8}{2} = 4m$$

8.Sol: If the soldiers while crossing a suspended bridge march in steps, the frequency of marching steps of soldiers may match the natural frequency of oscillation of the suspended bridge. In that situation resonance will take place, then the amplitude of oscillation of the suspended bridge will increase enormously, which may cause the collapsing of the bridge.

SOUND WAVES

1.Sol: Sound waves are mechanical waves

2.Sol: Sound waves require material medium to travel. As there is no atmosphere (Vacuum) on the surface of moon, therefore the sound waves cannot reach from one person to another.

3.Sol: The velocity of sound in a gas is directly porportional to the square root of its absolute temperature $\left(\text{as } v = \sqrt{\frac{\gamma RT}{M}}\right)$.

Since temperature of a hot day of more than cold winter day, therefore sound would travel faster on a hot summer day than on a cold winter day.

4.Sol: Speed of light is greater than that of sound, hence flash of lightning is seen before the sound of thunder.

5.Sol: Sound wave required medium.

CLASS XI PHYSICS KVPY(SA)-16

PREVIOUS YEAR QUESTIONS

1. An earthen pitcher used in summer cools water in it essentially by evaporation of water from its porous surface. If a pitcher carries 4 kg water and the rate of evaporation is 20 g per hour, temperature of water in it decreases by ΔT in two hours. The value of ΔT is close to (ratio of latent heat of evaporation to specific heat of water is $540°C$)

(a) 2.7°C (b) 4.2°C (c) 5.4°C (d) 10.8°C

2. Which of the following is NOT true about the total lunar eclipse?

(a) A lunar eclipse can occur on a new moon and full moon day.

(b) The lunar eclipse would occur roughly every month if the orbits of earth and moon were perfectly coplanar.

(c) The moon appears red during the eclipse because the blue light is absorbed in earth's atmosphere and red is transmitted.

(d) A lunar eclipse can occur only on a full moon day.

3. Many exoplanets have been discovered by the transit method, wherein one monitors a dip in the intensity of the parent star as the exoplanet moves infront of it. The exoplanet has a radius R and the parent star has radius 100R.If I_0 is the intensity observed on earth due to the parent star, then as the exoplanet transits,

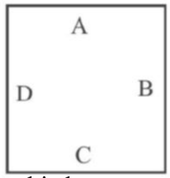
(a) The minimum observed intensity of the parent star is 0.9 I_0

(b) The minimum observed intensity of the parent star is 0.99 I_0

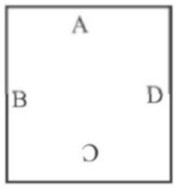
(c) The minimum observed intensity of the parent star is 0.999 I_0

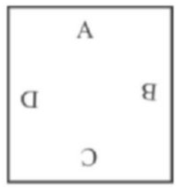
(d) The minimum observed intensity of the parent star is 0.9999 I_0

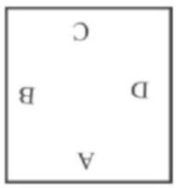
4. Letters A, B, C and D are written on a cardboard as shown in the figure

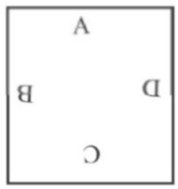


The cardboard is kept at a suitable distance behind a transparent empty glass of cylindrical shape. If the glass is now filled with water, one sees an inverted image of the pattern on the cardboard when looking through the glass. Ignoring magnification effects, the image would appear as

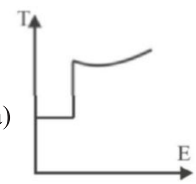
(a) 

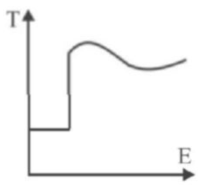
(b) 

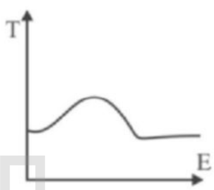
(c) 

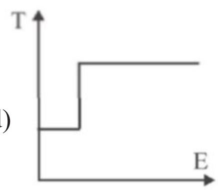
(d) 

5. A potential is given by $V(x) = (x+\alpha)^2/2$ for $x<0$ and $V(x) = k(x-\alpha)^2/2$ for $x>0$. The schematic variation of oscillation period (T) for a performing periodic motion in this potential as a function of its energy E is

(a) 

(b) 

(c) 

(d) 

ANSWER KEY

1. c **2.** a **3.** d **4.** d **5.** d

HINTS & SOLUTIONS

1.Sol: Rate of vaporization of water $= 20\frac{g}{hr}$

Water vaporized in $2\,\text{hr} = 40g = \frac{40}{1000}kg$

Given that $\frac{L}{C} = 540°C$

Heat looses by water = Heat gained for vaporization

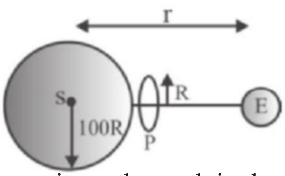
$$(m - \Delta m)C\Delta T = \Delta mL$$

$$(m - \Delta m)\Delta T = \Delta m\frac{L}{C}$$

$$\Delta T = \left(\frac{0.04}{(4 - 0.04)}\right)540 = -5.4°C$$

2.Sol: A lunar eclipse can occur only on the night of a full moon.

3.Sol: When the exoplanet reaches the line that joins the star and earth then energy (power) emitted by the star will be absorbed by the planet P. So the intensity of radiation that reaches the earth decreases.



The initial intensity at the earth in the absence of the planet is

$$I = \frac{P}{4\pi r^2}$$

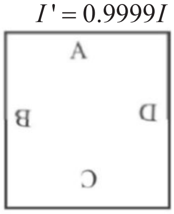
The intensity at the earth in the presence of planet is

$$I' = \frac{P'}{4\pi r^2} = \frac{0.9999P}{4\pi r^2}$$

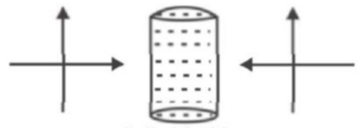
The effective area of the sun that emmits radiation is $4\pi(100R)^2 - 4\pi R^2$

$$I' = 0.9999I$$

4.Sol:

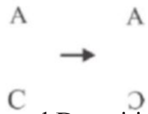


When an object is seen through a water filled cylinder then inversion takesplace along horizontal direction and no inversion takes place along vertical direction as shown in the figure



Cylindrical lens

A and C on vertical line so they are not inter changed but A gets horizontally reversed, which can't be recognized. C gets horizontally inter changed, as shown in the figure



Similarly B and D positions get interchanged but they are reversed horizontally as shown in the figure

So the correct image is (d).

5.Sol: Given that

$$v = \frac{1}{2}(x+\alpha)^2 \text{ for } x < 0$$

Acceleration of the particle is

$$\frac{d^2x}{dt^2} = -\frac{dV}{dx} = -(x+\alpha)$$

$$\Rightarrow \quad \frac{d^2(x+\alpha)}{dt^2} = -(x+\alpha)$$

The above equation represents SHM with time period

$$T_1 = \frac{2\pi}{\omega} = 2\pi \sec$$

T_1 is independent of energy

$$V = \frac{1}{2}k(x-\alpha)^2 \text{ for } x > 0$$

Acceleration of the particle is

$$\frac{d^2x}{dt^2} = -\frac{dV}{dx} = -k(x-\alpha)$$

$$\frac{d^2}{dt^2}(x-\alpha) = -k(x-\alpha)$$

The above equation represents SHM with time period

$$T_2 = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k}}$$

T_2 is independent of energy

The correct graph should be (d)